Proceedings of the 18th IFIP TC7
Conference on System Modelling and Optimization,
Detroit, Michigan, July 1997.

CONTROL OF TURBULENT FLOWS

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July 18, 1997

Abstract. It is useful for many industrial applications to be able to control turbulence in fluid flows, either to reduce it or, in some cases, to increase it. Active or passive control procedures are of interest. The problems that we face here are considerable and encompass those related to the control of complex nonlinear systems and those related to the direct numerical simulation of turbulent flows.

Our aim in this lecture is to report on some recent results obtained by the authors in an interactive collaboration between mathematicians and fluid dynamicists, and which represent a small step in the solution of this problem; this includes the mathematical modelling of such control problems, theoretical results (existence of optimal control, necessary conditions of optimality) and the development of effective numerical algorithms.

Introduction

It is useful for many industrial applications to be able to control turbulence in fluid flows either to reduce it or, in some cases, to increase it. At this time the most important applications are probably those arising in aeronautics which include the reduction of skin friction drag and the delay of transition to turbulence or separation of the boundary layer. Other important applications may be found in thermohydraulics, magnetohydrodynamics, climate and pollution forecasting. In combustion the objective is to increase turbulence for a better mixing of the fuel and its oxidant.

As in other control problems, such problems need first to be modelled, deciding (choosing) what is costly and what are the objectives; passive and active controls are of interest, and more recently robust control: the objective may be e.g. the design of an airfoil including its surface (shape optimization/passive control) or the active control of small actuators on the surface of the airfoil to properly respond to the coherent structures of the nearwall turbulence.

Key words and phrases. Fluid mechanics, turbulence, optimal control, robust control.

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The difficulties of the problem are considerable and much remains to be done. The description of the "state" of the system amounts to the resolution of the 3D Navier-Stokes equations in a turbulent context; the capacity of the present computers allows the calculation of such flows in simple cases, but this still demands much from the largest available computers in terms of computing power and storage requirements. As solutions to these nonlinear problems must be sought iteratively, we must numerically solve such problems several times, further compounding the computational expense of this procedure.

During the past years, a number of articles have appeared in the engineering and mathematical literatures concerning the control of turbulent flows and treating different aspects of the problem; see e.g. F. Abergel and R. Temam (1990), M. Gunzburger, L. Hou and T.P. Sovobodny (1990), H. Choi, P. Moin and J. Kim (1994), S.S. Sritharan (1991) and the references therein. The presentation which follows is mainly based on the article of F. Abergel and R. Temam (1990) hereafter referred to as [AT] which sets the problem of controlling turbulence in the framework of control theory in the spirit of J.L. Lions (1968), and on two articles under completion: T.R. Bewley, P. Moin and R. Temam (1997b), T.R. Bewley, P. Moin, R. Temam and M. Ziane (1997) hereafter called [BMT] and [BMTZ]; see also T.R. Bewley, P. Moin and R. Temam (1996), (1997a).

This article is organized as follows. In Section 1 we describe the modelling of the open loop control problem under consideration and give the main theoretical result. In Section 2 we describe the numerical algorithm which has been used without theoretical justification and which has produced a nearly ideal result in some cases in which we obtain an almost complete relaminarization of the flow. In Section 3 we discuss some other issues, namely some conjectures on the theoretical justification of the algorithm which we used, the wall information problem, and some preliminary remarks on the utilization and implementation of robust control.

Although the results obtained in Section 2 are quite significant (nearly optimal), we would like to emphasize that we are still far from practical (industrial) applications with several respects: the geometry is simple, the Reynolds number not too high, full information (and not just wall information) has been used; the practical implementation of the optimal control is not available and extensive calculations have been used which might be difficult to reproduce in real time. Nevertheless there is hope to obtain in the future, for more involved and more realistic problems, a still very useful if not as significant reduction of the cost function.

1. The Channel Flow Problem

We consider the flow of an incompressible fluid in a three dimensional channel as a simplified form of the flow in a wind tunnel. The channel occupies the region $\Omega = (0, \ell_1) \times (0, \ell_2) \times (0, \ell_3)$. The flow is maintained by an unspecified pressure gradient $P = P(\tau)$, in the x_1 (streamwise) direction. The flow will be controlled by the normal velocity of the upper wall Γ_w, $\{x_2 = \ell_2\}$.

Hence, the governing equations are the Navier-Stokes,

$$\begin{aligned} &\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla) u + \nabla p = P e_1 \quad \text{in } \Omega \times (0, T), \\ &\nabla \cdot u = 0 \quad \text{in } \Omega \times (0, T). \end{aligned} \tag{1.1}$$

Here $u = (u_1, u_2, u_3)$, function of x and t is the velocity vector; the pressure is $p(x,t) - x_1 P(t), p, P$ unknown, P accounting for the pressure gradient ($e_1 = (1,0,0)$). Periodicity is assumed for u and p in the direction x_1 and x_3 and

$$u = \varphi \text{ on } \Gamma_w \times (0,T) \tag{1.2}$$

whereas $u = \varphi$ on Γ_ℓ the rest of the lateral boundary of Γ. Finally the flux is fixed and given

$$\iint\limits_{x_1=0} u_1\, dx_2\, dx_3 = F \tag{1.3}$$

The weak formulation of (1.1)-(1.3), consists in looking for $u = u(x,t)$ which satisfies (1.2), (1.3) and

$$\frac{d}{dt}\int_\Omega uv\, dx + \int_\Omega \{\nu \nabla u \cdot \nabla v + [(u\cdot\nabla)u]\} dx = 0, \tag{1.4}$$

for every (smooth) test function v such that

$$\begin{gathered}\nabla \cdot v = 0, \quad v = 0 \quad \text{on } \Gamma_w \text{ and } \Gamma_\ell \text{ and} \\ \iint\limits_{x_1=0} v_1\, dx_2\, dx_3 = 0\end{gathered} \tag{1.5}$$

(see e.g. R. Temam (1984) for many related examples).

Now in the language of control theory, φ is the *control*, $u = u_\varphi$ is the *state* of the system, and the *state equation* consists of (1.2)-(1.5).

For the modelling of the control problem, we need to choose/define the cost function J. It consists of two terms $J = J_0 + J_1$. The first term, e.g.

$$J_0(\varphi) = \frac{\ell^2}{2}||\varphi||_X^2 = \frac{\ell^2}{2}\int_0^T \int\limits_{\Gamma_w} |\varphi|^2\, dx_1\, dx_3\, dt,$$

accounts for the cost of the control. The second term, e.g.

$$\begin{aligned} J_{1a}(u) &= \frac{1}{2}\int\limits_0^T \int\limits_\Omega |\operatorname{curl} u|^2\, dx\, dt, \\ J_{1b}(u) &= \int_0^T \int_{\Gamma_w} \frac{\partial u_1}{\partial x_2}\, dx_2\, dx_3, \\ J_{1c}(u) &= \frac{1}{2}\int_\Omega |u(x,T)|^2\, dx; \end{aligned}$$

represents the flow quantity (related to turbulence) which we want to minimize; J_{1a} was used in [AT] for the theoretical study and J_{1b}, J_{1c} are used for the computations in [BMT], J_{1b} representing the terminal value of the turbulent kinetic energy (TKE) and J_{1c} the time-averaged value of the drag.

The corresponding control problems now read (for $i = a, b$ or c):

$$\inf_{\varphi}\{J_0(\varphi) + J_{1i}(\varphi)\} \tag{1.6}$$

Omitting here certain theoretical questions addressed in [AT], the following results were essentially proved in [AT]:

The control problem (1.6) has a solution, corresponding to the (optimal) control $\bar{\varphi}$ and corresponding state $\bar{u} = u_{\bar{\varphi}}$. (1.7)

The optimal state $\bar{\varphi}$ satisfies the necessary condition of optimality

$$J_0'(\bar{\varphi}) + J_{1a}'(\bar{\varphi}) = 0,$$

described as usual by an equation for the adjoint state

$\bar{w} = w_{\bar{\varphi}}$ *(see [AT]).* (1.8)

The gradient algorithm and conjugate gradient algorithm converge to the optimal control $\bar{\varphi}$ if the initialization φ_0 belongs to a small neighborhood of $\bar{\varphi}$ in the space X. (1.9)

See [AT] for the details in the case of J_{1a}; the proof easily extends to J_{1c}. The proofs for J_{1b} are not available, but this cost function has been selected for its numerical (computational) simplicity, its physical interest and the engineers' conviction that drag cannot become negative in physically realistic flows.

2. Numerical Simulations

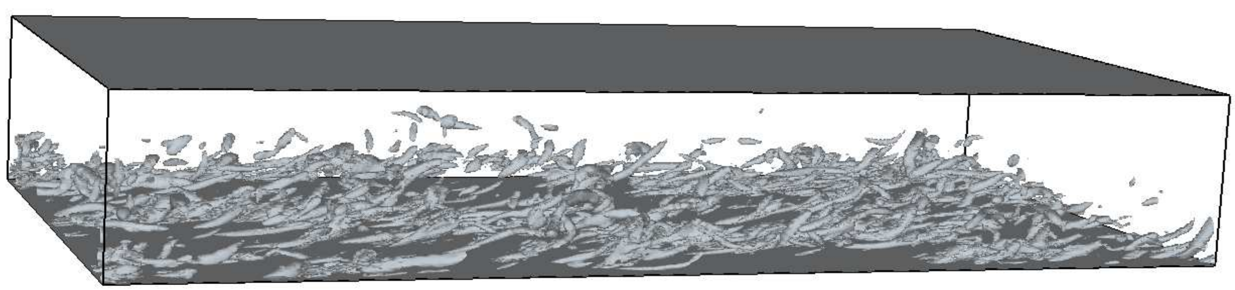


FIGURE 1. Coherent structures of a turbulent flow at $Re_\tau = 180$.

As motivation to the present work, we show in Figure 1 the coherent structures which appear near the wall in a turbulent channel flow and which we want to annihilate. The figure corresponds to a Reynolds number $Re_\tau = 180$, for which optimally controlled results are still under preparation; the results below correspond to $Re_\tau = 100$.

Taking into account present computational capabilities the algorithms provided by (1.9) are not feasible at this time for large T. Indeed, for the flow to attain some statistical equilibrium, T needs to be sufficiently large and (1.9) implies the resolution of the turbulent

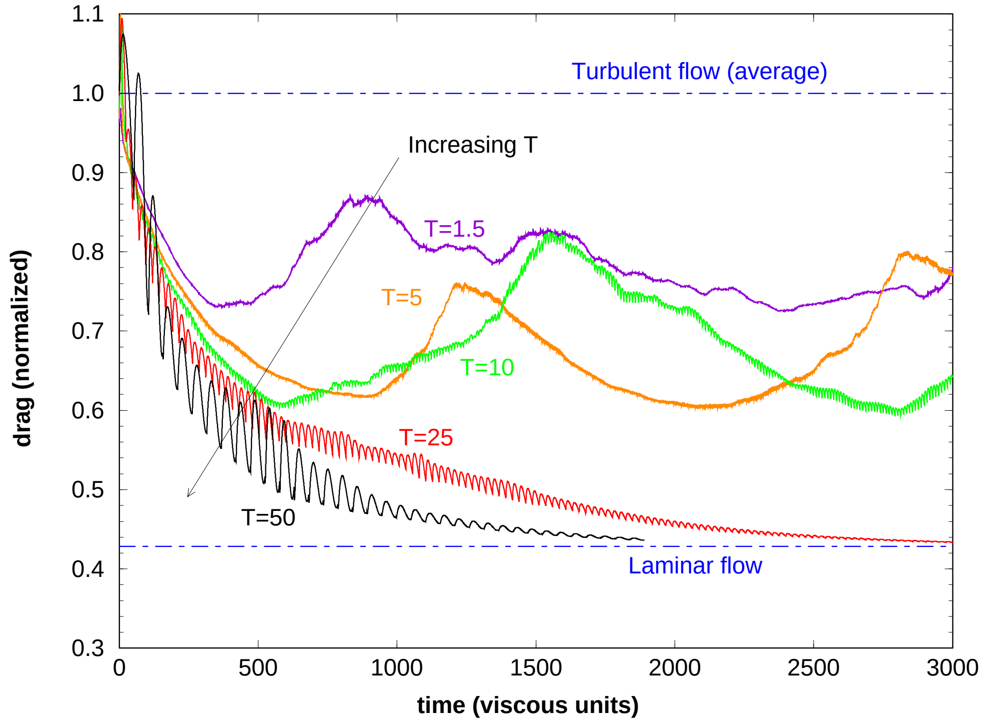


FIGURE 2. Time Evolution of the drag for different values of τ (denoted T in the figure).

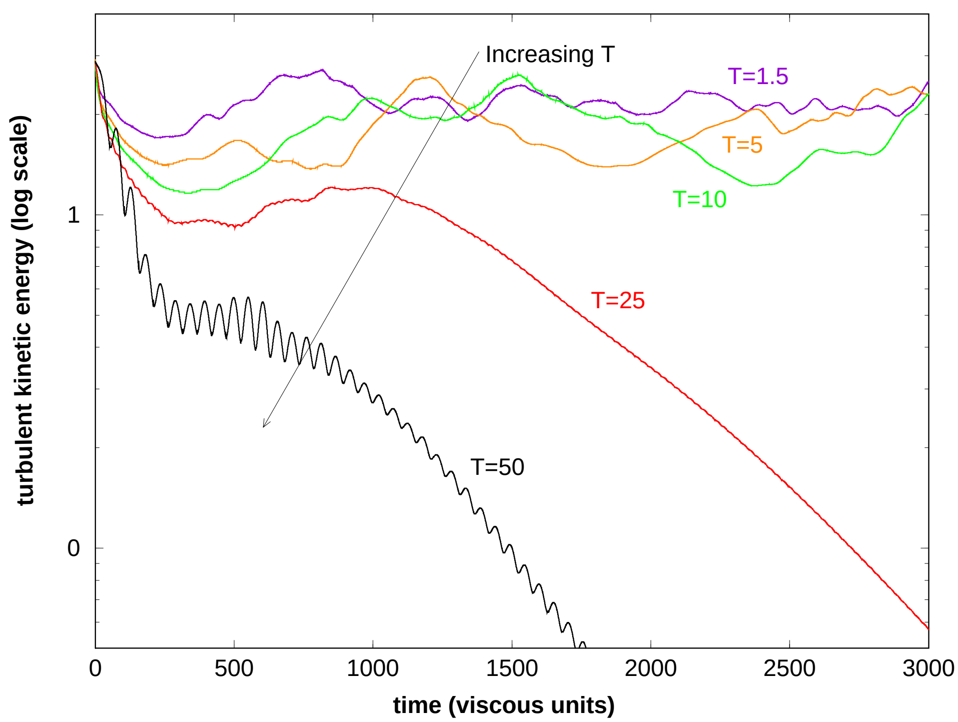


FIGURE 3. Time Evolution of the turbulent kinetic energy for different values of τ (denoted T in the figure).

Navier-Stokes equations and its adjoint on $(0, T)$, a number of times corresponding to the gradient iterations. Hence, our objective now, will be to look for suboptimal procedures which reduce the cost function, even if not making it minimal.

A very simple feedback law was used as a first step, which produced a drag reduction of approximately 17%. Note that the same procedure was used by H. Choi, R. Temam, P. Moin and J. Kim (1993) for the simpler case of the stochastic Burgers equation driven by a white noise; in this case the reduction of the cost function was approximately 75%.

We looked then for a multi-time-step procedure more adapted to the problem. Indeed there are here two different evolution equations which need to be solved, namely the Navier-Stokes equation itself and the abstract evolution equation of which the gradient algorithms can be seen as a time discretization:

$$\varphi^n - \varphi^{n-1} + \rho J'(\varphi^{n-1}) = 0 \qquad \Longleftrightarrow \qquad \frac{\partial \varphi(s)}{\partial s} + J'(\varphi(s)) = 0.$$

These two equations produce two different time constants and demand different time steps corresponding to different CFL (Courant-Friedrichs-Lewy) stability conditions.

Briefly described the algorithm implemented in [BMT] consists in dividing the interval $(0, T)$ into intervals of length τ; then on each interval $(m\tau, (m+1)\tau)$, we solve the control problem described in Section 1. At $m\tau, u$ is continuous $(u(m\tau + 0) = u(m\tau - 0))$, but however for the gradient algorithms, it appeared best to start the iterations with $\varphi^{m,0} = 0$ instead of $\varphi^{m,0} = \varphi(m\tau - 0)$. On each interval $(m\tau, (m+1)\tau)$ the Navier-Stokes equations are discretized with a time step $\Delta t << \tau$.

The evolution of the drag plotted in Figure 2 for different values of τ, at Reynolds number $Re_\tau = 100$, shows a near relaminarization for both $\tau = 25$ and $\tau = 50$ (in viscous time units). For such nearly relaminarized flows, the drag equals about 42 percent of its uncontrolled fully turbulent value (line —·—), which is the best we could achieve using the current approach. The evolution of the turbulent kinetic energy plotted in Figure 3 versus τ shows even more strikingly the relaminarization process initiated for $\tau = 25$ and $\tau = 50$ (in viscous time units). For both these plots, the cost functional used is the terminal value of the turbulent kinetic energy (i.e. $J = J_0 + J_{1c}$). The reason for the lobed behavior of the curves is that the penalty in the cost functional is only on the terminal value of the TKE (i.e., at the end of each optimization interval)—excursions of greater TKE during the middle of each optimization interval are allowed if they lead to reduced values of TKE by the end of the interval. The reader is referred to [BMT] for the details of the calculation and for physical insights which may be drawn from the results.

3. Other issues

(i) Theoretical justification of the algorithm.

Turbulent flows are believed to be statistically stationary and therefore the infinite time horizon, $T \to \infty$, is of physical relevance. In fact stationarity and ergodicity (if proven) imply that the time averages

$$\frac{1}{T} \int_0^T u(x,t)\,dt$$

converge, as $T \to \infty$, to a measure $\mu = \mu_\varphi$ which depends only on φ; see C. Foias and R. Temam (1975). Hence we could consider a minimization problem of the form,

$$\inf_{\varphi}\{\iint_{\Gamma_w} \frac{\partial u_1}{\partial x_2}\, dx\, d\mu_\varphi(u) + \frac{\ell^2}{2}[[\varphi]]^2\}$$

(where the norm $[[\cdot]]$ needs to be properly defined), and then compare the gradient algorithm applied to this optimization problem to the procedure described before. For the Burgers equation theoretical issues are addressed in G. DaPrato, A. Debussche and R. Temam (1994) who study the stochastic Burgers equations and in G. Da Prato and A. Debussche (1997) who address the control problem and its relation with the Hamilton-Jacobi equation.

(ii) Wall information.

The previous numerical study was based on the assumption that we have full information on the flow (u known everywhere), which, of course, is not realistic. In practice there will be wall sensors measuring certain quantities at the wall and the control algorithm should be therefore based on wall information only.

Two fundamentally different types of partial-information feedback controllers may be considered for such a purpose. In the first approach, the available flow measurements are fed back through a simple convolution kernel K to compute the control. The problem to be solved here is simply to find the best K which minimizes the flow quantity of interest.

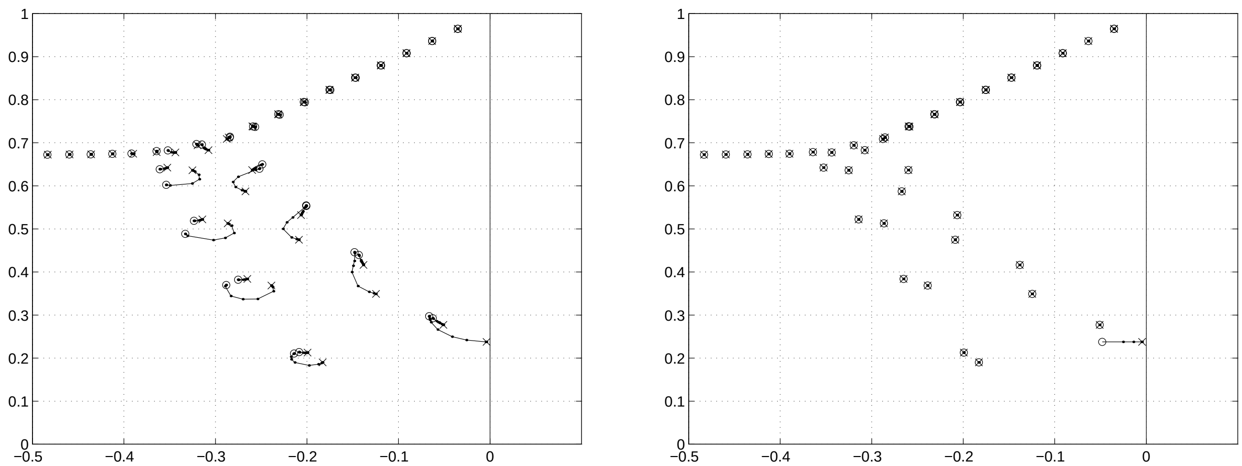
In the second approach, the available flow measurements are fed back through a simple convolution kernel L to compute a forcing term to update the state of an estimator, which is a set of equations which model the evolution of the flow itself. For the sake of analysis, the model equations may be taken simply as the Navier Stokes equation acting on some state estimate $\hat{u}$; however, this approach is particularly attractive from the standpoint that it can maximally utilize simple low-order models, such as POD-based models, for the state estimation of the near-wall turbulence. As an accurate state estimate is developed, the entire state estimate is fed back through a simple convolution kernel K to compute the control. The problem to be solved here is two-fold: i) to find the best L such that the state estimate is an accurate approximation of the state of the flow itself, at least near the wall where the measurements are made and the control is applied, and ii) to find the best K which minimizes the flow quantity of interest.

As discussed in T. Bewley, P. Moin and R. Temam (1997c), we may exploit the fact that we know the equations governing the flow to propose a *computationally expensive* adjoint-based technique to optimize the unknown convolution kernels K and L in these systems, which can only be performed on a supercomputer. However, once optimized, the feedback control rules themselves are *much simpler* than the adjoint-based technique used to optimize K and L, and thus may be considered for use in the laboratory.

(iii) Robust control.

Robust control for linear problems is well understood (see, e.g., J.C. Doyle et al., 1989 and K. Zhou, J.C. Doyle, and K. Glover, K. 1996)). Thus, when $(u \cdot \nabla)u$ is dropped or linearized around a stationary laminar flow solution, standard robust (i.e., H_∞) control techniques may be applied. This problem is addressed in a fluid-mechanical context in T.

Bewley, S. Liu and R. Agrawal (1997), where H_∞ control is used to stabilize a laminar flow to inhibit transition to turbulence. As shown in Figure 4, robust control focuses the control effort on the most unstable mode of the system, not "wasting" control effort on controllable but stable modes of the system. By using less control, there are fewer ways the control can "go wrong", and the system exhibits improved robustness to disturbances with less control effort than corresponding optimal (i.e., H_2) controlled system.



H_2 control (root locus vs. ℓ) *H_∞ control (root locus vs. γ)*

FIGURE 4. Movement of closed-loop eigenvalues versus control parameters ℓ and γ for H_2 (optimal) and H_∞ (robust) controllers applied to the Orr-Sommerfeld equation for $Re = 10,000$, $k_x = 1$, $k_z = 0$ using blowing/suction as the control variable.

The extension of the robust control concept to nonlinear problems such as turbulence is addressed in T.R. Bewley, P. Moin and R. Temam (1997a) from a computational perspective and in [BMTZ] from a theoretical perspective. In fact, it boils down to the optimal approach described previously with an additional forcing term χ added to the RHS of the Navier-Stokes equation (1.1) governing the system. The cost function considered is

$$J = J_1 + \frac{\ell^2}{2} \int_0^T \int_{\Gamma_w} |\varphi|^2 \, dx_1 \, dx_3 \, dt - \frac{\gamma^2}{2} \int_0^T \int_{\Omega} |\chi|^2 \, dx_1 \, dx_2 \, dx_3 \, dt.$$

The cost J is *minimized* with respect to the control φ, while simulataneously it is *maximized* with respect to the disturbance χ, in the spirit of a noncooperative game. Thus, the control φ is designed to handle that disturbance χ which is, in some manner, a *worst case* aggravation to the closed-loop system. By so doing, the control found is effective in the presence of a broad class of disturbances.

CONCLUSION

In this article, we have presented the modelling of a typical control problem in fluid flow, namely, the reduction of the turbulence in a channel. Despite the large size of the system (up to 2.4×10^7 state variables in Ω and 6×10^4 control variables on Γ_w), we have successfully implemented a control procedure which reduces the drag nearly to its absolute minimum, corresponding to laminar flow. Such a relaminarization of this flow by wall-normal blowing and suction has not been possible using *any* other control algorithm.

Further work in this direction will include: a) an attempt for theoretical justification of the algorithm, using probably the stationarity of turbulence, b) the optimization of more practical feedback control algorithms which are computationally inexpensive and depend on wall information only, and c) the design of robust controllers which account for worst-case disturbances in their derivation.

Acknowledgements

This work was supported in part by the National Science Foundation under Grants NSF-DMS-9400615 and NSF-DMS-9705229, by the Office of Naval Research under grant NAVY-N00014-91-J-1140, by the Air Force Office of Scientific Research under Grant F49620-93-1-0078, and by the Research Fund of Indiana University. The computer time was provided by NASA-Ames Research Center in support of this project.

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