Feedback control of turbulence

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A brief review of current approaches to active feedback control of the fluctuations arising in turbulent flows is presented, emphasizing the mathematical techniques involved. Active feedback control schemes are categorized and compared by examining the extent to which they are based on the governing flow equations. These schemes are broken down into the following categories: adaptive schemes, schemes based on heuristic physical arguments, schemes based on a dynamical systems approach, and schemes based on optimal control theory applied directly to the Navier-Stokes equations. Recent advances in methods of implementing small scale flow control ideas are also reviewed.

CONTENTS

INTRODUCTION

Many important advances have been made in the past decade in the field of turbulence control; recent reviews include: Bandyopadhyay (1986), Bushnell and McGinley (1989), Blackwelder (1989), Fiedler and Fernholz (1990), and Gad-el-Hak (1989, 1993). This paper will concentrate on one aspect of this subject: active feedback control of turbulence.

Active control schemes refer to methods which add energy to a flow, such as unsteady wall transpiration or the prescribed motion of an actuator. These are in contrast to passive techniques, which modify a flow without unsteady external input. Passive techniques include the placement of longitudinal grooves (riblets) on a surface to reduce the drag caused by turbulence (Choi *et al*. 1993b, Walsh 1990) and the use of compliant walls which deform in response to the overlying flow to stabilize a laminar boundary layer (Riley *et al*. 1988).

The external energy added in an active control scheme may be determined in advance (in which case the control scheme is termed open-loop or feedforward) or coordinated with real-time measurements of the flow itself (termed closed-loop or feedback control). The periodic forcing of a round jet (Lee and Reynolds 1985) to produce bifurcation (splitting into two jets) or blooming (expansion to a wide spray of vortex rings) and the hydrodynamic Lorenz forcing of an electrolytic fluid (Nosenchuck and Brown 1993) to restructure flow perturbations in the near wall region are excellent examples of effective open-loop control configurations in turbulent flows. However, in cases in which the control must interact with a specific set of turbulent fluctuations already present in the flow, such as the coherent structures, the random aspect of these structures reduces the effectiveness of an open-loop configuration. In these cases, we seek a "feedback control law" to relate measurements of the state of the turbulence in the flow to the resulting distribution in space and time of the control energy. It is this mathematical relation between what is sensed and what control is applied which will be systematically discussed in this paper. The feedback referred to in this context should not be confused with the feedback of information caused by the upstream influence of events which take place downstream through the flow itself, as discussed by Ho and Huerre (1984).

Adam Smith in *The Division of Labour* (1776) recalls:

> In the first fire-engines, a boy was constantly employed to open and shut alternately the communication between the boiler and the cylinder, according as the piston either ascended or descended. One of those boys, who loved to play with his companions, observed that, by tying a string from the handle of the valve which opened this communication to another part of the machine, the valve would open and shut without his assistance, and leave him at liberty to divert himself with his play fellows.

In the present study, our valves are the actuators, the points on the "machine" that we have access to tie to are the sensors, and our string is the feedback control law. As the resourceful boy, we seek the best arrangement of this string.

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The importance of coherent motions in turbulent flows (Robinson 1991) provides a physical target for active turbulence control schemes. Through feedback, control effort may be coordinated to manipulate these structures. This can have a profound overall effect on the turbulence. Herein lies the mathematical challenge of feedback turbulence control: in the midst of the vast range of spatial and temporal fluctuations of turbulence, identify those unstable coherent structures responsible for the regeneration of the turbulence and the most efficient distribution of control energy to achieve a desired effect.

Large-scale flow management schemes, which sense the gross flow features and then alter fixed or slowly-varying set points of the flow (*e.g.* the air-fuel ratio in a combustor) in order to optimize some combination of parameters, are well developed. One application is IC engine control (Amstutz *et al.* 1993), where the quantity of fuel injected may be actively updated using a linear quadratic regulator to maintain stoichiometry and thereby reduce pollutants and improve efficiency. Another example is combustion optimization (Padmanabhan *et al.* 1993), where the parameters for various open-loop actuators (*e.g.* speakers and vortex generating jets) are slowly altered using an optimization algorithm to simultaneously minimize r.m.s. pressure fluctuations and maximize volumetric heat release. It is the subject of this paper to study current progress in a technologically more challenging problem: small scale manipulation of the turbulent fluctuations themselves.

Feedback control of turbulence has been incorporated for millennia by birds and fishes. Adaptations range from structural modifications, such as bird feathers and shark denticles which protrude under certain flow conditions to become drag-reducing vortex generators (Bushnell and Moore 1991), to behavioral modifications, such as the active turning of the head of a schooling fish into the direction of cross-flow created by the tail of the fish ahead to align the boundary layer flow with its body and thereby reduce drag (Lighthill 1993). Man has only recently gained enough understanding of turbulence to effectively alter it through such an active fashion. There have been some mathematical developments in this field in the last few years to make a review of this nature appropriate and timely. We will attempt to survey how these new schemes fit in with others currently being investigated.

This paper will categorize current methods of feedback control by examining their mathematical dependence on the equations governing the flow phenomenon to be controlled. The first category to be reviewed is adaptive schemes, which attempt to develop models and controllers for turbulent fluctuations via some learning algorithm without regard for the detailed flow physics. We then touch upon schemes based on heuristic physical arguments, which have found some success in flows in which the coherent motions are qualitatively understood. The dynamical systems approach is also described. In this approach, turbulence is decomposed into a small number of representative modes and then the dynamics of these modes are examined to determine appropriate control schemes. Finally, schemes based on optimal control theory applied directly to the Navier-Stokes equations are discussed. In these schemes, the desired quantity to be minimized is written mathematically as a cost function and then this cost function is minimized in the space of the control by appealing directly to the equations governing the flow itself.

Several devices have recently been developed which may be used to detect and force turbulent flows; a cursory review of these is given in the final section of this paper.

Linear stability theory (Drazin and Reid 1981) can be quite useful when considering flow control problems. For example, in some flow configurations, all growing disturbances convect downstream from their source, in which case the flow is said to be convectively unstable. This is in contrast to the case in which some of the growing disturbances can travel back upstream and continually disrupt the flow even after the initial disturbance is neutralized, which is referred to as absolute instability. In configurations which are convectively unstable, active control schemes applied near the point where perturbations originate can be especially effective.

Receptivity issues (Goldstein and Hultgren 1989 and Hill 1993a), which are closely related to the stability problem, can also provide some guidance for the design of control laws by identifying where stable and unstable modes are most easily modified in a particular flow (Hill 1992). Though related to the control problem, space does not permit review of stability and receptivity issues in this manuscript.

FEEDBACK CONTROL SCHEMES

Adaptive schemes

Schemes belonging to this class perform system identification and controller determination without regard for the dynamics known to take place in the flow or the Navier-Stokes equations known to govern these dynamics. These schemes all use some adaptive method which, through adjustment of parameters by the engineer or via optimization by the scheme itself, tune the parameters of the feedback control law to achieve satisfactory results. The control algorithm may be based on feedback control theory for linear dynamic systems, linear or non-linear adaptive networks, or other standard techniques which have been developed and used in a wide variety of disciplines. These schemes are effective for simple control configurations. However, for configurations with multiple noisy sensors and complex flow dynamics, the ability of adaptive schemes to converge to efficient control algorithms is reduced.

If a system may be approximated as responding linearly to control input, a very wide range of techniques may be used to build a controller, including root locus analysis, Bode design techniques, and linear quadratic regulators (Franklin *et al.* 1991). Kwong and Dowling (1993) provide a good example of the effective use of such techniques to design a feedback controller to reduce the unsteadiness and improve the pressure recovery in a diffuser. By performing a frequency sweep with an actuator and plotting the resulting response at a sensor downstream, the effective (linear) transfer function of the flow configuration was deduced; this process is termed system identification. With this system model, a controller was designed using Bode analysis techniques. Note that systems which are dominated by non-linear effects can not be analyzed or effectively controlled using these linear transfer function techniques (Choi *et al*. 1993a). Modified techniques, though cumbersome, can be used to account for system nonlinearities in this framework (Graham and McRuer 1961, Gelb and Vander Velde 1968).

Candel (1992) investigated the use of a linear adaptive algorithm called the Least Mean Squares (LMS) method (Isermann *et al*. 1992) to optimize the parameters of a feedback control arrangement in a combustor. The control arrangement was rather simple: one sensor was connected through a simple transfer function to one actuator, and a second sensor was used to monitor the performance of this feedback system. The parameters of the transfer function were then varied according to the LMS algorithm to maximize some performance measure based on the output of the monitoring sensor. Note that this type of system is only capable of representing linear relationships. To control flow effects which themselves develop (approximately) linearly, however, such as the early stages of development of Tollmien-Schlichting waves in a boundary layer (and a wide range of other phenomena), this is the most appropriate adaptive scheme to use, as it's convergence properties are better than those of non-linear adaptive algorithms.

Jacobson and Reynolds (1993a) and Fan *et al*. (1993) have investigated the use of non-linear adaptive algorithms called neural networks for the purpose of flow control. These schemes relate the outputs of the sensors to the inputs of the actuators through a non-linear function with variable coefficients and sigmoid saturation functions. An update scheme called the back-propagation algorithm has been developed which allows update of these coefficients based on error measures in a manner similar to the LMS algorithm for the linear configuration above. With a sufficient number of terms, this non-linear configuration can represent complex control laws; however, hand tuning of adaptation parameters is needed to achieve good convergence properties. Many improvements on the standard neural network configuration are possible to achieve better convergence properties (Hertz *et al*. 1991); these techniques should be exploited as much as possible when implementing non-linear adaptive algorithms for best results.

Schemes based on physical arguments

In situations in which the dominant physics is well understood, judgment can guide an engineer to design effective control schemes. Success is limited, however, by the engineer's understanding of the physical processes involved; in the case of turbulence, our understanding is still limited despite several decades of intense research.

A good example of the use of physical understanding in the design of flow control schemes is the active cancellation of disturbances. Thomas (1990) reviews several independent investigations of active cancellation schemes applied to delay the onset of transition in boundary layers. Most of these experiments investigated the production of disturbances designed to counter the effects of artificially produced wave-like disturbances introduced further upstream; this is a considerably easier problem than dealing with the fleeting disturbances which appear naturally. Through the tuning of the amplitude and phase of a feedback control law, several of the schemes successfully delayed the transition that was caused by the upstream disturbances.

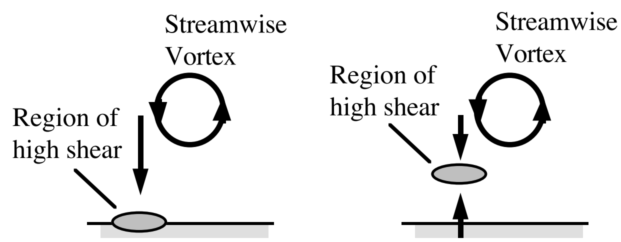

Figure 1. An active cancellation scheme applied to turbulent flow, from Choi *et al*. (1994).

An active cancellation scheme was used by Choi *et al*. (1994), to reduce the drag in a fully-developed turbulent flow by mitigating the effect of the near-wall vortices. By opposing the near-wall motions of the fluid, which are caused by the near-wall vortices, with an opposing wall control as shown in Figure 1, the high shear region was lifted away from the wall. A direct numerical simulation of this scheme applied to turbulent channel flow demonstrated about 20% drag reduction when the control was chosen to oppose the vertical motion at $y^+ = 10$. The $y^+ = 10$ sensing location achieved the best results. When this sensing location was moved above $y^+ = 25$ and the same control scheme was used, the flow response to the control became unbounded. Sensing the instantaneous normal velocity at $y^+ = 10$ is, of course, very impractical. It is highly desirable to confine both sensing and actuation to the wall, as discussed later in this manuscript. Thus, Choi *et al*. computed the correlation of quantities measurable at the wall with the normal velocity above the wall. Surprisingly, the wall pressure did not exhibit a high correlation with the normal velocity. Using a Taylor series expansion and the equation of continuity, they obtained an expression relating the normal velocity at a point near the wall to the instantaneous wall shear. However, using this expression to estimate the normal velocity away from the wall resulted in only a 6% drag reduction. This is comparable to the drag reduction that can be achieved with simpler, passive means such as riblets.

Schemes based on dynamical systems

The tools of dynamical systems theory have proven useful in analyzing and interpreting turbulence dynamics (Aubry *et al*.

1988). Due to their large range of spatial and temporal scales, turbulent flows are known to have relatively high dimensions in this framework even at fairly low Reynolds numbers, which makes analysis of these systems quite difficult (Keefe *et al.* 1992). However, there has been some instructive work in representing the dynamics of coherent structures in boundary layers with systems of much lower dimension using the proper orthogonal decomposition (POD) method (Aubry *et al.* 1988, Berkooz *et al.* 1992, 1993). This method provides a set of eigenfunctions which are particularly efficient in representing second order turbulence statistics with a small number of modes. The algebraic complexity of the equations governing the turbulence is increased when expressed in this modal form.

As an example of how a low-order decomposition of a turbulent flow can be used for control, let us reconsider the reduction of drag in a wall-bounded flow. In the dynamical systems framework, the movement of the coherent structures may be represented locally as the orbiting of a low dimensional state (perhaps 10 to 20 modes) around several unstable fixed points; the passage of one set of coherent structures leads to a jump in the state to a different unstable orbit, or to a different distribution of coherent structures (Bloch and Marsden 1989a,b). Through the action of control, the unstable orbits around some of these fixed points may be stabilized. When this is achieved, some of these stabilized orbits will have more desirable qualities than others. The goal is then to converge to the nearest stabilized orbit with desirable qualities, in this case, to an orbit with low drag.

Research in the development of control schemes based on this method of stabilizing the attractors of a low-dimensional approximation of a turbulent chaotic system is still in progress. Keefe (1993a,b) has investigated the use of the OGY method (of Ott, Grebogi, and Yorke 1990) for feedback control of turbulence with limited control energy.

The OGY method is illustrated in Figure 2. The state of the turbulence is represented schematically as the intersection of a stable manifold and an unstable manifold. The desired trajectory moves along the path shown. If control could be applied to put the state of the flow onto the stable manifold *exactly* and there were no noise in the system, the state would converge toward the desired trajectory and no further control would be needed. However, noise in the system caused by the unmodeled dynamics of the flow prohibits this, and thus the state always has a tendency to wander off in the unstable directions. Thus, control effort must be applied to move the state towards the stable manifold to stabilize the system to the desired orbit. Note that this is more efficient than applying control to move the state towards the desired trajectory itself, as no control forcing is needed in the stable directions. Keefe (1993b) suggests applying an intermittent control forcing, shutting off the control when it is within a prescribed tolerance from the stable manifold. Though attempting to make a certain orbit completely stable would require excessive amounts of control energy, partial stabilization of orbits with desirable qualities could reap some benefits.

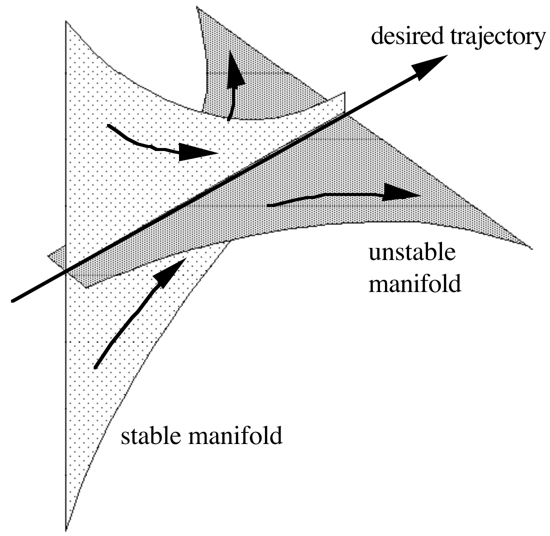

Figure 2. The OGY method, from Keefe *et al.* (1993b).

As a simple example illustrating this technique, consider balancing a baseball in the center of a saddle on the back of a standing horse. No matter how calm the horse, there will always be some motion which will tend to perturb the baseball. According to the OGY method, control need only be applied to get the baseball back up on the centerline of the horse (in the most direct manner possible) because the shape of the saddle itself will tend to keep the ball centered between the horse's head and his tail. This example just has one mode (the location of the baseball), one stable direction (front/back), and one unstable direction (left/right), with perturbations caused by unanticipated motions of the horse. A dynamical system modeling a turbulent flow, in comparison, will have several interdependent modes and several stable and unstable "directions" (referred to collectively as manifolds), with perturbations, perhaps quite large, caused by the unmodeled dynamics of the flow. Further complicating the flow control problem, the desired flow dynamics are not stationary, and the shape of the saddle which describes the dynamics of the modes is quite complex.

Optimal control schemes

The above schemes, though exhibiting varying degrees of success for the purposes for which they were designed, fail to provide us with a rigorous theory to determine the most efficient feedback control law for a given flow control problem. Application of optimal control theory directly to the equations of motion governing the flow itself, the Navier-Stokes equations, provides this rigorous framework for flow control (Abergel and Temam 1990). With optimal control schemes, we have a systematic method with which we may derive feedback control laws for the most efficient distribution of control effort to achieve various desired effects.

The seminal idea of the optimal control method is the minimization of a cost functional which is written to represent the physical problem of interest. Minimization of this functional is achieved by computing the gradient of the cost functional in the space of the control through an adjoint formulation, then updating the control with a gradient algorithm. For an unsteady problem such as turbulence, the cost functional is usually a time-averaged quantity; an unsteady control is found which minimizes this functional over some finite interval of time. This control, over the entire time interval under consideration, is iteratively updated using a gradient scheme until the best solution is reached.

To make the optimal method practical for implementation, certain approximations have been made (Bewley *et al.* 1993), the most significant of which is that the optimization is performed considering only very short-time developments of the flow. We shall call this the suboptimal approximation. The suboptimal method is now illustrated by example.

An example

Consider again the problem of the reduction of drag in a turbulent channel via small amounts of wall transpiration (blowing and suction) at the lower wall. The notation used in this discussion is that (u,v,w) represent the velocities in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively. The control applied to the wall normal velocity is represented by ϕ such that the boundary conditions on the controlled wall are: $u = 0$, $v = \phi\,(x,z)$, $w = 0$. The integral of ϕ over the wall is taken to be zero so that there is no net mass flux through the wall.

An instantaneous cost functional $J(\phi)$ may now be written to represent the balance of the quantities that we want minimized, which we take to be a linear combination of the drag integrated over the wall and a measure of the net control effort, which in this case is taken to be the mean square value of the control. We thus write the instantaneous cost functional† for this problem as

$$J(\phi) = \int\int_w \frac{\partial u}{\partial y}\, dx\, dz + \frac{l}{2}\int\int_w \phi^2\, dx\, dz,$$

where l is a weighting factor which represents the expense of applying the control (a number which is small if the control is cheap and large if it is expensive). We desire to find the control ϕ which minimizes this cost functional some time shortly in the future (which we write as $t = t^{n+1}$) based on current observations of the flow (at $t = t^n$). Note that the first term on the RHS has an implicit dependence on the control ϕ, which is applied for the time duration (t^n, t^{n+1}). We can compute the "optimum" value of ϕ to minimize $J(\phi)$ at $t = t^{n+1}$ using a gradient descent algorithm if the gradient of $J(\phi)$ in the space of the control ϕ is found. This gradient information may be found by considering the adjoint of the Navier-Stokes equations (Bewley *et al.* 1993).

To compute the gradient of the cost functional in the space of the control, we begin by writing the Fréchet differential (Vainberg 1964) of the cost functional $J(\phi)$ as

$$\frac{DJ(\phi)}{D\phi}\phi' \equiv \lim_{\varepsilon\to 0} \frac{J(\phi+\varepsilon\phi')-J(\phi)}{\varepsilon}$$
$$= \int\int_w \frac{\partial}{\partial y}\left(\frac{Du}{D\phi}\phi'\right) dx\, dz + l\int\int_w \phi\,\phi'\, dx\, dz.$$

The differential is the gradient of the cost $J(\phi)$ in the space of the control ϕ taken in some arbitrary control direction ϕ'. Through adjoint calculus (Greenberg 1971), it is a straightforward process to re-express the integrand in the first term on the RHS as a term which multiplies ϕ', which results in

$$\int\int_w \left(\frac{DJ(\phi)}{D\phi} - Re\,\pi - l\,\phi\right)\phi'\, dx\, dz = 0,$$

where Re is the Reynolds number and π is the adjoint pressure on the wall, which comes out of the solution of an adjoint differential equation, which itself depends on the state of the flow and thus must be solved at every time step. As the control distribution ϕ' is arbitrary, we may now extract the desired expression for the gradient (Vainberg 1964):

$$\frac{DJ(\phi)}{D\phi} = Re\,\pi + l\,\phi.$$

With this gradient information, the control may be updated according to a gradient algorithm such as

$$\phi^{n,k+1} = \phi^{n,k} - \mu\,\frac{DJ(\phi^{n,k})}{D\phi},$$

where k is the iteration index at each time step.

Using direct numerical simulation, the method in the example above has been proven effective in turbulent channel flow, giving a 17% drag reduction via small amounts of wall-normal blowing and suction (Bewley *et al.* 1993). This computation used the idealization that all the turbulent fluctuations above the wall were known and could be accounted for when computing the adjoint pressure π. By approximating the velocities near the wall with a Taylor series expansion from the wall, the adjoint problem may be cast in a form which, with further approximations, may be solved analytically (Hill 1993b). Preliminary computations of this type of scheme result in a 15% drag reduction, showing that performance is not significantly degraded by making these approximations (Bewley *et al.* 1993). Further, these approximate schemes are algebraically much simpler to implement, as they don't require the on-line solution of an adjoint differential equation at each time step.

Gradient Algorithm

In the discrete case, the gradient algorithm used in the above example has a clear physical analogy. Consider for the moment a configuration with only two control points; let us call them ϕ_1 and ϕ_2. The value of the cost function‡ will vary depending on how these control jets are set; we seek the global minimum of the cost function in the domain created by these control variables. As this domain is searched, one might find that the cost function forms a surface something like a bowl, as shown in Figure 3.

† Note that the integrand in the first term of $J(\phi)$ is not purely positive. It has been found (Bewley *et al.* 1993) that this is not essential for an effective scheme; rather, the cost functional should be written to accurately represent the desired control objective.

‡ The word function is used to denote dependence on a discrete field, whereas the word functional denotes dependence on a continuous field.

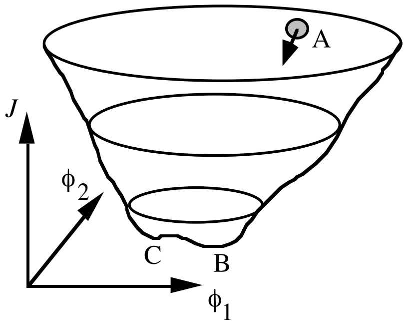

Figure 3. Representation of a possible shape of the cost function in the space created by two control jets ϕ_1 and ϕ_2.

Starting from point A in Figure 3, optimal control theory provides information on the local shape of the bowl, as indicated by the shaded region, including the direction of maximum decrease of the cost function, indicated by the arrow. By continually moving in the direction of this gradient, the simple gradient algorithm proceeds towards a minimum of the cost function. Note, however, that depending on where point A is relative to the minima, this algorithm may converge to the global minimum B or to some other local minimum such as C; this is a drawback of searching with a gradient routine. Mathematical analysis of optimal turbulence control problems led Fattorini and Sritharan (1992) to conclude that the existence of local minima of the cost function, and thus non-unique solutions to the optimal control problem, must in general be expected. As the flow develops in time, the shape of the bowl changes, and the optimal control scheme attempts to track the movement of the minimum point.

For a cost function with a long, narrow valley leading to its minimum, the simple gradient scheme used above can get stuck bouncing from one wall of the valley to the other without proceeding to turn directly down the valley towards the minimum point. In such cases, the conjugate gradient method has proven to be much more efficient. This method proceeds in a direction which is a linear combination of the direction of maximum decrease of the cost function and the direction used in the previous step. Thus, the scheme retains a momentum term that helps turn the descent path to proceed down narrow valleys. The application of this gradient scheme to optimal flow control problems is currently being investigated.

Optimal versus suboptimal schemes

The method illustrated by the example above is formally called a *suboptimal* method in the language of control theory, as it looks only one short time step into the future. A truly *optimal* method attempts to minimize the time averaged value of $J(\phi)$ over some finite time interval T, which is closer to the desired effect (in the above example, the desired effect might be to reduce the total fuel consumption on a particular excursion). Note that the suboptimal method does not look ahead to anticipate further development of the flow, and thus the solution by the suboptimal method does not necessarily correspond to the solution by the optimal control method, and is an approximation to the desired control objective.

The differences in complexity between the optimal and suboptimal schemes described above may be realized by drawing an analogy to a computer algorithm to play chess. A suboptimal chess program looks ahead one step to determine the move that leaves as good a position on the board as possible. Similarly, a suboptimal turbulence control scheme looks ahead one time step to determine the set of control velocities that leaves as good (*i.e.* low) a value of the cost functional as possible at the next time step. An optimal chess program, on the other hand, investigates all possible developments of the game a certain number of steps into the future (knowing how the other player may respond), and then moves in the direction that leads to the best final position on the board. Similarly, an optimal turbulence control scheme investigates all possible developments of the flow a certain amount of time into the future (knowing approximately how the flow will respond), and then applies the set of control velocities that leads to the best (*i.e.* lowest) time-averaged cost functional. Such a method requires significantly more resources than the suboptimal method. The implementation of truly optimal control schemes to the Navier-Stokes equations are currently being investigated.

Extension to other control problems

A strength of the optimal control method is that it may be easily generalized to a wide variety of control problems. Other flow effects may be minimized by replacing the drag term in the cost functional with a different term of physical interest. For example, to reduce the flow noise contamination of sonar systems, one would like to reduce the pressure fluctuations on the skin of the submerged vessel (Bewley and Moin 1994). The cost functional may be altered accordingly

$$J(\phi) = \int\int_w p'^2 \, dx \, dz + \frac{l}{2} \int\int_w \phi^2 \, dx \, dz,$$

and the derivation of the control scheme follows as before. More physically relevant terms representing the expense of the control may also be implemented in a straightforward fashion.

In addition to different cost functionals, different flow forcing techniques may also be analyzed with the optimal control method. For example, instead of using a wall-normal boundary condition as the control, one might add a right hand side forcing term in the Navier-Stokes equations, representing electromagnetic control forcing of near wall fluid (Hou and Ravindran 1993) through a configuration similar to that of the TFM tiles of Nosenchuck and Brown (1993). The technique used to analyze problems with interior forcing such as this is similar to that used to analyze problems with boundary control (Choi *et al.* 1993a).

Discussion

For very simple feedback configurations with just a handful of sensors and actuators, adaptive schemes have been shown to perform well. However, as the configurations get more complex, allowing the possibility of more effective manipulation of the flow, the most effective feedback control law also becomes more complex. In such cases, the performance of adaptive schemes degrade and become highly dependent upon the method of training; we are thus motivated to base the control law on the flow physics.

This physical basis of a feedback control law may be made intuitively or mathematically. An intuitive control law has the attractive property that it is usually quite simple (*e.g.* countering the near wall vertical motion of the fluid with an opposite wall control). However, its performance is limited by the engineer's understanding of the flow phenomenon to be controlled.

For the purpose of designing a mathematical control law, it is logical to make a low order model of the turbulent structures and then to design a control scheme based on an examination of the dynamics of this model. The theoretical development of the best model to use for this is currently under investigation. Low order models of the flow have been used to analyze the dynamics of the coherent structures present in the flow. This interpretation of the turbulent structures allows identification and possible stabilization of orbits with desirable properties in this low dimensional model. Thus, control may be applied to encourage the desired dynamics of the coherent structures. The identification of the flow state with a limited number of noisy sensors may be performed rigorously using a non-linear estimator (Sritharan 1993, Anderson and D'Souza 1994).

At the opposite end of the scale from adaptive schemes, with optimal control schemes one explores various possible developments of the flow over some time interval (using the Navier-Stokes equations and assumed complete information of the initial flow state) and then mathematically arrives at the optimum set of control velocities over this interval to minimize a given time-averaged cost functional. However, such schemes require a) very large computer resources, and b) full information about the exact state of the flow at the initial time. The problem may be made tractable by two approximations: i) by considering the development of the flow only a short time into the future, so that the problem may be solved in a single time step (the suboptimal approximation), and ii) by approximating the fluid motions near the wall by an extrapolation of the flow quantities measured with wall mounted sensors.

The approximation of the optimal problem, then, leads to a set of control velocities directly, bypassing the identification of known structures near the wall. This can be seen as both a strength and a weakness. It results in a straightforward set of equations (the adjoint problem) that may be approximately solved analytically, producing an inexpensive sensor-output to actuator-input transfer function that may be easily implemented. The approach is easily generalized to several types of flow problems. However, the short time approximation has the implication that the control scheme constantly attempts to drive the cost function down. In the long run, this might not always be the best solution. As it is sometimes useful to sacrifice a pawn in chess for long-term gain, it might also be helpful to consider the further development of the flow, perhaps via a low-order dynamical systems approximation, rather than always seeking "instant minimization." Thus, it is not clear at this point which of the several avenues currently being investigated will finally prove to be the most successful and the most general, and further research is strongly motivated.

IMPLEMENTATION ISSUES

As illustrated in our introductory metaphor, any feedback control system consists of three components: the valve, the tie down points, and the string—in the language of control theory: actuators, sensors, and the control law. After the above discussion of various ideas, ranging from simple to quite elaborate, on how to arrange the string, we will now review recent developments of the devices to which the ends of this string may be tied.

The most notable advance in the past few years in the area of implementing turbulence control ideas has been the emergence of Micro Electro Mechanical Systems (MEMS) technology, which employs the methods developed for the fabrication of silicon chips to construct very small mechanical devices (Wise 1991). Miniaturization of this scale for both sensors and actuators is necessary for feedback control of turbulence due to the very small scales of the coherent structures in high Reynolds numbers flows of engineering interest. New questions arise in the fabrication of flow devices in silicon, which makes this an active area of current research (see recent Proceedings of the IEEE MEMS Workshops for several examples). Researchers are currently attempting to miniaturize several of the devices reviewed herein using MEMS technology.

Methods of sensing

Two desirable attributes of flow sensors are that they be robust and that they don't significantly disrupt the flow. For these reasons, most practical sensors for active flow control are flush mounted on a wall. At a wall, we may measure both skin friction and wall pressure.

For situations in which the wall pressure is important (for instance, in control schemes designed to reduce flow-induced noise), there are a plethora of devices, essentially small microphones, which have been developed for measuring pressure fluctuations. One example by Cho *et al.* (1989) is a capacitive pressure sensor built with a small flexible membrane. Note, however, that it has been found (Choi *et al.* 1994) that pressure is not a good indicator for detecting and controlling the sweep and ejection events which accompany near-wall coherent structures in wall-bounded flows.

Using a Taylor series extrapolation, the near wall flow may be estimated directly from shear stress measurements on the wall, though these estimates are only valid fairly near the wall (Choi *et al.* 1994). For the purpose of shear stress measurement at a wall, several types of sensors have been investigated recently; we will review four of the most popular: floating element sensors, piezo-electric foils, hot films, and surface acoustic wave (SAW) sensors.

A floating element sensor consists of a small rectangular patch of silicon supported by thin beams (Haritonidis 1988). The flow over the device exerts shear forces on the patch and the resulting stresses in the supporting beams may be measured by various methods, including the differential measure of capacitance (Schmidt *et al*. 1988) and an active electrostatic re-balancing technique with a comb actuator (Jaecklin *et al*. 1992). This device is quite attractive because it directly measures shear stress in a way that doesn't interfere with the flow. However, the yield in the fabrication of these devices is still quite low (about 10%) due to the intricacies in their design. Further perfection of design and manufacturing techniques may make this device quite promising.

Piezoelectric foils consist of thin films of polyvinylidene-fluoride (PVDF) coated with a very thin layer of aluminum (Nitsche *et al*. 1989). Portions of the PVDF films are crystallized, and the resulting artificial polarization exhibits a piezoelectric effect when subjected to normal and shear stresses. Detectors placed below the films then sense the field created by the piezo effect of the sheared crystal. The manufacture of these devices is simple and robust, but their sensitivity to both normal and shear stresses poses difficulties. To measure shear stress, Nitsche *et al*. propose placing two detectors side by side in opposite configurations so that, by combining the signals from the two sensors, the effects of the normal stress cancel and the resulting signal is proportional to the shear stress. However, the small magnitude of the signals being measured results in the familiar problem of losing the signal in the noise created by imperfect cancellation of the contributions due to the normal stress fluctuations.

A hot film sensor may be used as an indirect measure of skin friction by calibrating the heat transfer of the film as a function of the applied shear; however, care must be taken in this calibration, as the static and dynamic responses differ (Alfredsson *et al*. 1988) and the response is nonlinear. Such a sensor is easy to manufacture but difficult to use as it doesn't measure skin friction directly; if used underwater, a hot film sensor must be thermally coupled but electrically isolated from the flow, and the thermal cross talk from other sensors and/or actuators must be minimized—this would be difficult in a control configuration where sensors and actuators must be placed in close proximity.

Devices may also be built to measure the propagation speed of surface acoustic waves (Varadan *et al*. 1989, 1990). The surface wave propagation speed is a function of the stresses caused by the overlying flow; by building devices which measure the wave speed in alternate directions, one may estimate the instantaneous shear stress. These devices are quite sensitive and respond linearly to the shear stress; however, they are also sensitive to the normal stress, temperature fluctuations, electric noise, and drift of the resonant frequency of the oscillator circuit—any system using SAW devices must be able to account for these dependencies in a way that doesn't lose the signal in the noise created by imperfect cancellation in the differencing process.

Methods of actuation

Several ideas for the active manipulation of small scale turbulent structures near a wall are currently under investigation; below is a description of a few of the more popular configurations (Wilkinson 1990 discusses others).

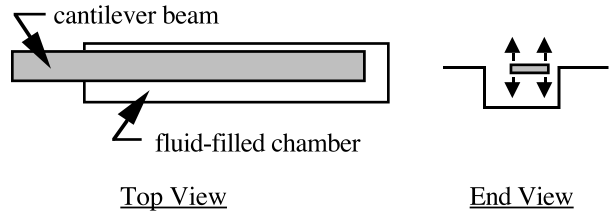

Figure 4. Beam-chamber configuration, from Jacobson and Reynolds (1993b).

Figure 4 illustrates a beam situated over a cavity which is allowed to passively fill with fluid from all sides. By vibrating the beam at its resonant frequency, it can be made to force the fluid out of the cavity. The vibration can be created by a piezoelectric effect (Wiltse and Glezer 1993 and Jacobson and Reynolds 1993b) or by periodic optical heating of one side of the beam (Lammerink *et al*. 1991). The output through the narrow gap is concentrated and directed primarily in the vertical direction. By taking advantage of the different flow patterns caused by upward and downward motions of the beam, a strong flow pattern may be established. It has been observed that the flow field created by such a device is a set of counter-rotating vortices centered over the narrow gap with common flow up; by modulating the vibration amplitude, the magnitude of this disturbance may be controlled. An advantage of this method is the strong flow field it can create; disadvantages are that this flow field is necessarily quite complicated and the beam itself is difficult to manufacture and rather fragile.

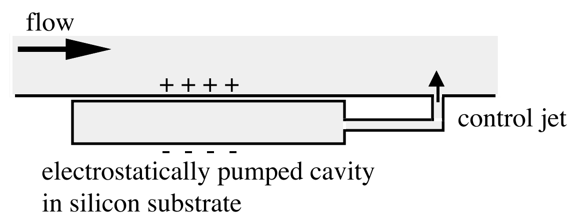

Figure 5. Electrostatically pumped cavity (side view), from Breuer (1993).

Electrostatic forces or conventional speakers may be used to pump the fluid inside a cavity (Breuer 1993, Meier and Zhou 1991, Weinstein and Balasubramanian 1977). In Breuer's configuration (Figure 5), the cavity leads to a small hole through which the excess volume of fluid must travel. This results in very precise blowing and suction applied through the actuation hole. Difficulties include the fragility of the membrane and the tendency of the membrane to short out by touching the lower wall of the cavity.

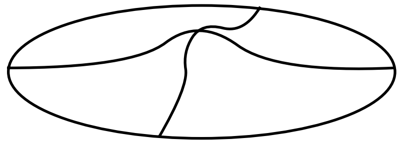

Figure 6. Variable bump, from Lumley (1989).

Figure 6 shows a bump on a surface which may be regulated by a piezoelectric material underneath a membrane (Lumley 1989). In Lumley's configuration, the bump is approximately Gaussian in shape; the flow field caused by the bump may be visualized as a horseshoe vortex with a common flow towards the wall immediately downstream of the device. One problem with this device is that getting sufficient displacement with piezo material is difficult. A different mechanism which alleviates this problem (but has a significant activation time) is a solenoid-activated valve leading to a high pressure source below the membrane (Breuer *et al.* 1989). A variant on this idea is to create the pressure below the membrane by the controlled phase transition boiling of a liquid—this avenue has yet to be thoroughly explored (Wise 1991). Bumps may also be activated by attaching magnets to the membrane and situating electromagnets below (Wilkinson and Balasubramanian 1985) or electrostatically forcing the membrane itself (Weinstein and Balasubramanian 1977). Variable bump devices have the potential of being more robust than cantilevered beams and pumped cavities; however, the perturbation to the flow field caused by the bump is complicated.

Differential wall heating is another possible method of control (Liepmann *et al.* 1982, Nosenchuck *et al.* 1987). Using wall heating to create the velocity fluctuations actually has several simultaneous effects, including the alteration of the specific volume of the heated fluid, associated buoyancy effects, and changes in viscosity, all of which should be accounted for by the control scheme. The idea is attractive from the robustness standpoint because it has no moving parts. However, for control of turbulence in practical applications, very high power heaters and very low thermal capacitance of the wall would be required to achieve the necessary frequency response. This might be most easily realized using a laser with optical fiber access to small metal patches on the surface. For a discussion of optical heating issues, see Lammerink *et al.* (1991).

Finally, if the fluid is electrolytic or can easily be made that way by addition of salts, hydrodynamic Lorenz forcing is another control option (Nosenchuck and Brown 1993, Tsinober 1990). The work of Nosenchuck and Brown, though open loop, is an excellent example of the effectiveness of this forcing technique. In their configuration, electric and magnetic fields are applied in the streamwise and spanwise directions with the use of well-positioned magnets and electrodes. By varying the electric field with an active control circuit, this configuration could be used in a feedback configuration. However, manufacturing such units on a scale small enough to interact actively with turbulent coherent structures might prove to be difficult.

Other considerations

Flow control is most effective when applied to critical flow regimes. As mentioned in the abstract, one method of finding these critical regimes is by a receptivity analysis of the flow under consideration. For example, the effects of a control scheme can be quite dramatic when applied near the transition point of a boundary layer flow, the separation point on an airfoil, or the nozzle of a jet, where flow instabilities magnify quickly. However, skin friction reduction by active control in a fully turbulent regime would be approximately proportional to the surface area covered by the actuators. For the reduction of skin friction drag over large surface areas, inexpensive, modular systems that are both robust and simple to diagnose and replace are the only alternative.

A modular configuration which has been proposed by Gad-el-Hak (1993) and Reynolds (1993) is to tile a portion of the surface of an airplane with sensor-actuator-controller units fabricated in silicon which can be mass-produced using MEMS technology. The layout of the sensors and the actuators in the tiles must be designed carefully, as the direction of the flow may not be known *a priori* and would change with flight conditions. Estimates on the requirements for control units under flight conditions have been computed by Wilkinson (1990) and Gad-el-Hak (1993). At the typical aircraft cruise conditions quoted by Gad-el-Hak (u_∞ = 300 m/s, u_τ = 10 m/s, ν = 3x10^{-5} m^2/s), the wall unit scale is ν/u_τ = 3 μm and the non-dimensional time unit is $\nu/u_\tau{}^2$ = 0.3 μsec. The average spanwise spacing of the streaky structures is about 100 wall units; a few sensors and actuators must span this gap in order to effectively counter the turbulent motion, implying actuators and sensors with widths on the order of 50 μm. The passage of coherent structures at these flight conditions (estimated by the time it takes a coherent structure to convect at $0.8{*}u_\infty$ a distance of 400 wall units) would be approximately once every 5 μsec. Power requirements are considered by Muntz *et al.* (1993). These guidelines give very rough estimates on the spatial density of sensors and actuators and the response time required in this implementation—production of control units on this scale with today's technology would be difficult.

CONCLUDING REMARKS

Current investigations down several different avenues of possible feedback control schemes provide the theoretical groundwork for future applications. More immediately, and of equal significance, current investigations of the feedback control of turbulence are enhancing our fundamental understanding of the mechanisms responsible for the maintenance and regeneration of turbulence itself.

The advent of Micro Electro Mechanical Systems technology for both sensors and actuators allows us to begin to consider practical implementations. Even with these advances, schemes which require the coverage of large surface areas with thousands of sensors and actuators are at present out of reach. The applications which are currently most promising for the implementation of feedback control schemes have critical areas where the flow is quite sensitive to modification, such as areas of separation or transition.

Returning to the metaphorical steam engine, simultaneous development of the string, the valve, and the tie-down points leave us optimistic about future developments in this field.

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