Relaminarization of $Re_\tau=100$ turbulence using gain scheduling and linear state-feedback control

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The first successful application of linear full-state feedback optimal control theory to consistently relaminarize turbulent channel flow at $Re_\tau=100$ with full state information and gain scheduling is reported. The actuation is zero-net mass-flux blowing and suction on the channel walls. Two key issues central to the success of this strategy are: (a) the choice of the mean-flow profile about which the equations are linearized for the computation of the linear feedback gains, and (b) the choice of an objective function which targets the control effort on the flow perturbations of interest. A range of mean-flow profiles between the laminar and fully turbulent profiles and a weighted energy measure which targets flow perturbations in the near-wall region were found to provide effective feedback gains. A gain-scheduling strategy to tune the feedback gains to the nonstationary mean-flow profile is introduced, resulting in consistent relaminarization of the turbulent flow in all realizations tested. [DOI: 10.1063/1.1608939]

Leveraging the linearized equations of fluid motion in an attempt to develop effective flow control algorithms is a fairly recent strategy which has rapidly become quite popular. One of the earliest studies of this type[1] evaluated the superposition control concept using the Orr–Sommerfeld equations with periodic blowing and suction as the boundary condition. Another early study[2] used heating and cooling at the wall coordinated with a simple proportional control scheme based on measurements of wall shear to modify the viscosity of the flow in order to suppress instabilities. The linearized equations have also been used[3] to evaluate the strategy now commonly known as ‘‘opposition control.’’[4] The behavior of a so-called ‘‘vorticity flux’’ scheme has been quantified[5] by computation of neutral curves for the controlled linear system.

Classical control theory has been applied to two-dimensional perturbations in a laminar channel flow using a streamfunction formulation of the Orr–Sommerfeld equation.[6] Blowing and suction actuation was computed using feedback of wall shear. Using a full-state-feedback integral compensator the flow could be stabilized significantly. Modern control theory has also been used to compute optimal ($\mathcal{H}_2$) controllers using this streamfunction formulation.[7] The same formulation has also been used to develop reduced-order robust (LTR) controllers for the multi-wavenumber case.[8] A similar approach was followed[9] to develop robust ($\mathcal{H}_\infty$) controllers, accounting for effects of localized actuation. These two-dimensional controllers may be extended for application to three-dimensional flows by augmenting an *ad hoc* scheme in the third dimension.[10] Applied to a turbulent channel flow at $Re_\tau=100$, this scheme resulted in a maximum drag reduction of 17%.

Three-dimensional perturbations have also been considered directly,[11] using both optimal ($\mathcal{H}_2$) and robust ($\mathcal{H}_\infty$) control strategies for both sub- and supercritical Reynolds numbers at isolated wavenumber pairs in a linearized channel flow. The key property making this work is the complete decoupling of the control problem at different wavenumber pairs when the Orr–Sommerfeld/Squire equations are used and all variables with spatial variation are Fourier transformed in the streamwise and spanwise directions. It was suggested[11] that the optimal control for the full physical system could be obtained through an inverse Fourier transform of optimal controllers computed via such a technique for a large array of wavenumber pairs. It was theoretically predicted[12] that such controllers, computed for a spatially invariant distributed system, should be spatially localized with exponentially decaying tails. Well-behaved localized control feedback kernels of this sort were first obtained for a Navier–Stokes system[13] using a slightly modified version of the problem formulation studied previously.[11] The performance of these linear controllers, using both full information and wall information only, has been thoroughly quantified in terms of their ability to prevent chanel-flow transition.[14]

Prior to the present work, nonlinear control strategies, in an expensive model predictive control framework, have been uniquely successful in relaminarizing fully developed channel-flow turbulence using blowing and suction as the method of actuation.[15] In this work, a receding-horizon optimization strategy is used, which means that a large time interval is divided into smaller subintervals, and then the control is optimized over these subintervals successively using an adjoint-based algorithm. It was found[15] that the performance of the resulting control can differ widely depending on the choice of flow properties penalized by the objective function, as also indicated by other studies.[16] A terminal measure of turbulent kinetic energy on each subinterval was found to be the most suitable choice to obtain relaminarization. The importance of choosing the right quantity of the flow to target in the optimization of a controller for a fluid flow is well known; direct numerical simulations[17] show that the linear coupling term [$\mathcal{C}$ in (2)] is crucial for the maintenance of the turbulence near the wall. It has been suggested[17] that an objective function targeting the effect of this coupling term could result in an effective controller. Inspired by this work, an energy weighting of the form $f(y) = 1 + U'(y)^2$ was introduced in the objective function in the present work [see (6)].

We now give a very brief summary of the control approach used in the present work, referring the reader to our earlier paper[14] for many of the details. The Orr–Sommerfeld/Squire equations are used as a model of the flow system. These equations are derived from the Fourier transform (in the x and z directions) of the Navier–Stokes equation linearized about a mean-flow profile $U(y)$, and may be written at each wavenumber pair $\{k_x, k_z\}$ as

$$\begin{aligned} \Delta \dot{\hat{v}} &= \{-ik_x U \Delta + ik_x U'' + \Delta(\Delta/\mathrm{Re})\}\hat{v}, \\ \dot{\hat{\omega}} &= \{-ik_z U'\}\hat{v} + \{-ik_x U + \Delta/\mathrm{Re}\}\hat{\omega}, \end{aligned} \tag{1}$$

where $\Delta \equiv \partial^2/\partial y^2 - k_x^2 - k_z^2$. The Reynolds number $\mathrm{Re} = u_\tau h/\nu$ parametrizes the problem, where h is the half-width of the channel, u_τ is the mean friction velocity of the uncontroller flow, and ν is the kinematic viscosity of the fluid. A Chebyshev collocation technique is used for the discretization in y at each wavenumber pair $\{k_x, k_z\}$. Boundary conditions are handled in the construction of the differentiation matrices in such a way that spurious eigenvalues are eliminated. Invocation of the homogeneous boundary conditions on $\partial\hat{v}/\partial y$ (resulting from the no-slip condition $\hat{u} = \hat{w} = 0$ at the wall and the continuity equation $ik_x\hat{u} + \partial\hat{v}/\partial y + ik_z\hat{w} = 0$) allows inversion of the Laplacian on the left-hand side of (1) and expression of (1) in matrix form:

$$\underbrace{\begin{bmatrix} \dot{\hat{\mathbf{v}}} \\ \dot{\hat{\boldsymbol{\omega}}} \end{bmatrix}}_{\dot{\hat{\mathbf{x}}}_f} = \underbrace{\begin{bmatrix} \mathcal{L} & 0 \\ \mathcal{C} & \mathcal{S} \end{bmatrix}}_{N} \underbrace{\begin{bmatrix} \hat{\mathbf{v}} \\ \hat{\boldsymbol{\omega}} \end{bmatrix}}_{\hat{\mathbf{x}}_f}, \tag{2}$$

where boldface denotes the vectors obtained via discretization. Control is applied via blowing and suction at the channel walls. A lifting technique is used to formulate the control equations in state-space form. To accomplish this, the flow perturbation is decomposed such that

$$\hat{\mathbf{x}}_f = \hat{\mathbf{x}}_i + \hat{\mathbf{x}}_h. \tag{3}$$

The inhomogeneous part $\hat{\mathbf{x}}_i$ is taken to satisfy the nonzero boundary conditions and a numerically convenient equation on the interior of the domain; in the present case, we construct the so-called "lifting" function $\hat{\mathbf{x}}_i$ to satisfy the simple equation $N\hat{\mathbf{x}}_i = 0$ on the interior at any instant. Assembling the controls (i.e., the values of $\hat{v}$ at the upper and lower walls) into a control vector $\hat{\phi}$, this system may easily be solved for arbitrary $\hat{\phi}$ and written as

$$\hat{\mathbf{x}}_i = Z\hat{\phi}. \tag{4}$$

The part $\hat{\mathbf{x}}_h$ therefore satisfies homogeneous boundary conditions, and the interior equation governing $\hat{\mathbf{x}}_h$ may be found by substitution of (3) into (2). Noting (4), the result may be written

$$\underbrace{\begin{bmatrix} \dot{\hat{\mathbf{x}}}_h \\ \dot{\hat{\phi}} \end{bmatrix}}_{\dot{\hat{\mathbf{x}}}} = \underbrace{\begin{bmatrix} N & NZ \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\mathbf{x}}_h \\ \hat{\phi} \end{bmatrix}}_{\hat{\mathbf{x}}} + \underbrace{\begin{bmatrix} -Z \\ I \end{bmatrix}}_{B} \underbrace{\dot{\hat{\phi}}}_{\hat{\mathbf{u}}}. \tag{5}$$

We have arrived at the desired state-space form. Note that the control $\hat{\mathbf{u}}$ is the *time derivative* of the normal velocity at the upper and lower walls, the state $\hat{\mathbf{x}}$ is the control $\hat{\phi}$ appended to the homogeneous vector $\hat{\mathbf{x}}_h$, and the state-space representation is decoupled in Fourier space at each wavenumber pair $\{k_x, k_z\}$. Note also that, for the convenient lifting function we have used here, we may take $NZ = 0$ in the above expression.

The magnitude of the flow perturbation is measured as a weighted integral of the square of the velocities over the flow domain. Rewriting this measure in $\hat{v}$–$\hat{\omega}$ form and introducing a weighting function $f(y)$ gives

$$\hat{E} = \frac{1}{8k^2}\int_{-1}^{1} f(y)\left(k^2|\hat{v}|^2 + \left|\frac{\partial \hat{v}}{\partial y}\right|^2 + |\hat{\omega}|^2\right)\mathrm{d}y = \hat{\mathbf{x}}_f^* Q \hat{\mathbf{x}}_f, \tag{6}$$

where $k^2 = k_x^2 + k_z^2$. Noting the decomposition (3), this measure may be written in terms of the state variable $\hat{\mathbf{x}}$ as

$$\hat{E} = \hat{\mathbf{x}}^* \begin{bmatrix} Q & QZ \\ Z^*Q & Z^*QZ \end{bmatrix} \hat{\mathbf{x}} \triangleq \hat{\mathbf{x}}^* \mathcal{Q} \hat{\mathbf{x}}. \tag{7}$$

We now seek the control $\hat{\mathbf{u}}$ which, with limited control effort, minimizes the weighted flow perturbation energy (6) on $t \in (0, \infty)$. This is a standard optimal control problem. Defining the objective function $J = \int_0^\infty (\hat{\mathbf{x}}^* \mathcal{Q} \hat{\mathbf{x}} + \ell^2 \hat{\mathbf{u}}^* \hat{\mathbf{u}})\mathrm{d}t$, the control $\hat{\mathbf{u}}$ which minimizes J is given by $\hat{\mathbf{u}} = K\hat{\mathbf{x}}$, where $K = -(1/\ell^2)B^*X$ and X is the positive-definite solution to the Riccati equation $XA + A^*X - (1/\ell^2)XBB^*X + \mathcal{Q} = 0$. Note that ℓ^2 is used as an adjustable parameter which scales the penalty on the control effort in the cost function, and that this penalty term is a function of $|\dot{\hat{\phi}}|^2$ in the present formulation. Due to the continuity of $\hat{v}$, excursions of $|\hat{\phi}|^2$ are penalized naturally in the $\hat{\mathbf{x}}^* \mathcal{Q} \hat{\mathbf{x}}$ term of the cost function; no additional penalty on $|\hat{\phi}|^2$ was found to be necessary in the present work.

The optimal control problem described here has been derived for each wavenumber pair $\{k_x, k_z\}$ independently. By assembling the corresponding physical space controller via an inverse Fourier transform, we may derive feedback con-

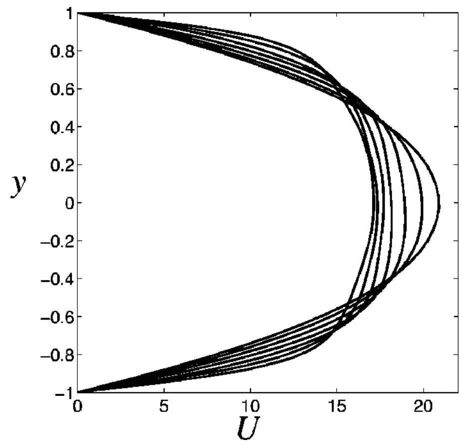

FIG. 1. Mean-velocity profiles used to compute the control kernels used in the gain-scheduled control strategy. The profiles were obtained by initializing a flow with simply the mean turbulent flow profile $U(y)$, then relaxing this mean profile back to the steady-state laminar profile using a nonlinear simulation code.

volution kernels that can be used to compute the control input in the physical domain. The convolution integral by which the control is computed in physical space is given by

$$\hat{\phi}_{\pm 1}(x,z,t)=\int_{\Omega}(k_{v,\pm 1}(x-\overline{x},\overline{y},z-\overline{z})\,v(\overline{x},\overline{y},\overline{z},t)$$
$$+k_{\omega,\pm 1}(x-\overline{x},\overline{y},z-\overline{z})\,\omega(\overline{x},\overline{y},\overline{z},t))\,d\overline{x}\,d\overline{y}\,d\overline{z}, \tag{8}$$

where the feedback kernels $k_{v,\pm 1}$ and $k_{\omega,\pm 1}$ are the inverse Fourier transform of the feedback gains on $\hat{v}$ and $\hat{\omega}$, respectively, which are computed on a large array of wavenumber pairs. The convolution kernels obtained for the flow considered here are spatially localized and similar to those reported earlier by our group.[13,18]

During the relaminarization of a turbulent flow, there is a significant change in the mean-flow profile. In order to approximate the dynamics of the turbulent flow system with a linear equation which models the system dynamics as accurately as possible, feedback kernels are computed based on the Orr–Sommerfeld/Squire equations linearized about a range of representative mean-flow profiles, including the laminar profile, the fully turbulent mean-flow profile, and several profiles in between, as depicted in Fig. 1. These control kernels are then used according to a so-called "gain-scheduled" scheme that implements linear control feedback based on those kernels computed for the mean-flow profile that most closely corresponds (in an L^2 sense) to a sliding time average (integrated over a period of 1 viscous time unit) of the current mean-flow profile in the simulation. This idea was first presented as an effective solution for the present application at the APS-DFD meeting in November 2000, by Högberg.

A benchmark problem has been set up for testing the effectiveness of the control algorithm in a constant-mass flux channel flow in a $4\pi\times 2\times 4\pi/3$ box with $64\times 82\times 64$ grid points. Several independent realizations of fully developed turbulence at $Re_\tau=100$ are used to initialize the flow state in

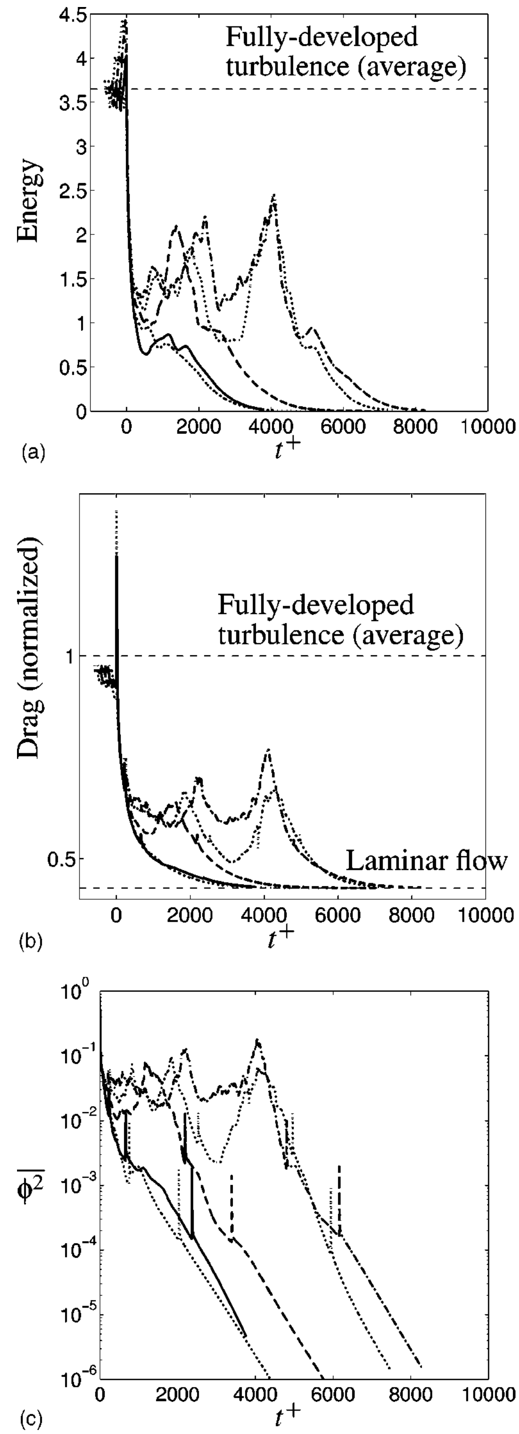

FIG. 2. Evolution of initially fully developed turbulence at $Re_\tau=100$ when a gain-scheduled linear controller computed using $\ell=0.1$ and $f(y)=1+U'(y)^2$ in (6) is applied to different initial conditions. The control is turned on at $t=0$ in all cases. Top: energy of flow perturbation. Middle: normalized total drag. Bottom: mean-square value of the control ϕ. Note that application of the gain-scheduled linear control feedback causes the fully turbulent three-dimensional flow to relaminarize in all flow realizations tested.

the tests of controller effectiveness; the results for each initial condition are both qualitatively and quantitatively similar, indicating the generality of the control effectiveness. One case at a higher resolution in the normal direction using 128 points was also tested to verify the results. The code uses a

Fourier discretization in x and z and second-order-accurate finite differences in y. The solver is implicit on all terms involving wall-normal derivatives to allow for strong blowing and suction without affecting the CFL restriction on the time step. The same code was used previously,[15] where the same flow was relaminarized using an adjoint-based optimization technique. The computation of the feedback convolution integrals was implemented both in physical space and in Fourier space. The two methods are equivalent and result in identical control signals, validating the correctness of the implementation. For the sake of computational efficiency in the direct numerical simulation, the Fourier implementation was used in the simulations presented here.

Five different simulations have been performed in order to test the efficiency of the control scheme. In all cases the turbulent flow has relaminarized, and perturbation energy and drag have been significantly reduced. Figure 2 shows the time history of the perturbation energy, normalized drag, and the energy of the control signal. Each time the gain-scheduling algorithm switches to a new set of kernels based on the evolution of the mean flow profile, there is a small transient bump in the control energy and in the drag. Note also the initial transient increase in the drag when the control is turned on. The magnitude of this transient varies between the cases considered and is approximately 30%–40%. The controlled simulations have been stopped once the perturbation energy level is sufficiently low that the subcritical flow relaminarizes once the control is turned off, as confirmed by subsequent (uncontrolled) DNS. The time needed to obtain relaminarization varies for the different realizations, and in some cases the perturbation energy decays monotonically while in other cases it does not. The gain scheduling approach introduced in this paper allows the controller to adapt to the varations in the mean-flow profile on a case-by-case basis. It should be noted that a majority of the initial conditions tested could also be relaminarized using only one set of control kernels computed for one of the intermediate mean velocity profiles depicted in Fig. 1. The gain scheduled approach, however, succeeded in relaminarizing the initially turbulent flow for *all* realizations of initial conditions tested.

The present paper has shown that modern linear control theory is useful for determining effective control strategies for fully turbulent flows via a technique which schedules the linear control feedback gains based on the mean flow profile. Interestingly, most of the states about which the governing equations were linearized in the present control computations were neither stationary solutions of the governing equations nor the desired target state. Instead, in order to approximate the dynamics of the turbulent flow system with a linear equation which models the system dynamics as accurately as possible, linearization was performed around mean flow profiles which were, in a sense, ‘‘close’’ to the mean-flow profiles encountered during the relaminarization process. The present investigation did not result in control designs that could consistently relaminarize turbulent channel flow without using gain scheduling, but that does not rule out this possibility.

The improved controller performance encountered when using a weighted energy measure in the objective function suggests that there might be better measures of the flow perturbations for model-based control strategies to focus on than just the perturbation energy.

The importance of linear mechanisms in transition and turbulence has been emphasized by many authors. The present work indicates that the information contained in the linearized equations is sufficient, at this Reynolds number, to design linear controllers that consistently relaminarize near-wall turbulence with actuation at the wall. For practical implementation of this flow control scheme, there is a need for a state estimator for the mean and fluctuating components of the flow. Motivated by the success of linear state feedback in the present full-information control problem, extended Kalman filters implementing linear measurement feedback into the corresponding estimation problem are currently being explored for this purpose, and will be reported elsewhere.

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