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Excitation design for damage detection using iterative adjoint-based optimization—Part 1: Method development

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ABSTRACT

A technique is developed to answer the important question: "Given limited system response measurements and ever-present physical limits on the level of excitation, what excitation should be provided to a system to make damage most detectable?" The solution is developed by forming an augmented system that is the union of the undamaged and damaged systems. The difference between measurable outputs of the undamaged and damaged systems then simply becomes a state in this augmented system which may be then maximized by maximizing a related and easily developed cost function. By formulating an adjoint version of the optimization problem, the gradient of this cost function with respect to the excitation may be calculated very efficiently, and a straight forward gradient ascent procedure follows. This process is demonstrated on a 2 DOF system with a nonlinear stiffness, where it is shown that an optimized excitation increases the detectability of the damage by several orders of magnitude.

1. Introduction

The field of structural health monitoring (SHM) seeks to assess the integrity of structures for the primary purpose of moving from time-based maintenance to a more cost effective condition-based maintenance strategy. Consequently, most approaches to SHM are non-destructive in nature. One common non-destructive approach is known as vibration-based SHM. In this approach, a structure is instrumented with an array of sensors at various locations. The structure is then excited and its dynamic response recorded. This response is then interrogated to extract features that are correlated with damage. A survey of the SHM literature [1,2] reveals that a great deal of attention has been paid to the data interrogation portion of the SHM process, with almost no attention paid to the excitation design. This focus is quite understandable in many applications where only ambient excitation is available, such as most civil engineering applications. However, there are many applications where the excitation is selectable (e.g., most wave propogation approaches to SHM), and, indeed, where proper excitation selection is essential. As a simple example, consider a beam or column with a crack that is nominally closed due to a preload. If the provided excitation is not sufficient to open and close the crack, the detectability of the crack in the measured output will be severely limited.

There have been a couple of results reported in the literature on the effect of various types of excitation. In [3], Peeters considers the effects of different excitation sources on the detectability of damage for civil structure applications, though no procedure for selecting the excitation is suggested. Alonso [4] uses an orthogonal wavelet decomposition to consider the

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effect of excitation frequency on detection accuracy, though again, no excitation design procedure is specified. There have also been a number of papers that seek to design a good excitation. Nichols [5] presents a novel excitation design technique based on using the output of a chaotic attractor as the input to the system under consideration. Olson [6,7] expands on this work by selecting attractor parameters via a genetic programming approach. In [8], Vanhoenacker employs a set of carefully chosen sinusoids for excitation which are shown to elucidate certain types of nonlinear behavior. Giurgiutiu [9] uses a lamb wave excitation via an embedded piezoelectric actuator. While all four of these references present a means of specifying (and in certain cases optimizing) an excitation, they all place significant restrictions in the form that the excitation can take.

This paper presents a method for designing excitations for the purpose of damage detection. As will be seen, the developed technique places almost no restrictions on the excitation, other than physically motivated restrictions on excitation signal amplitude, duration, and sampling frequency. This method is applicable to a very broad class of nonlinear (and linear) systems and seeks to answer a very important question: "Given limited system response measurements and ever-present physical limits on the level of excitation, what excitation should be provided to a system to make damage most detectable?"

The developed technique is conceptually quite simple. It seeks to find an excitation that maximizes the difference between the outputs of the damaged and undamaged system. A reasonable approach to this (and many) optimization problems is a gradient ascent. However, because every point of the excitation time series is a free variable, the optimization problem is one in a very high dimensional space, and the gradient is very computationally expensive to evaluate via a finite difference method. This difficulty is avoided by formulating an adjoint version of the optimization problem, where it is found that this gradient may be calculated very efficiently.

The remainder of this paper presents the proposed method, followed by an example involving a nonlinear 2 DOF structure. A discussion of the strengths and weaknesses of the proposed method as well as future research plans is then given.

2. Main result

The concept of damage only makes sense when the current system state is compared to an undamaged state [10]. All damage detection strategies require either a model for or data from at least the undamaged structure. For classifying anything beyond the existence of damage, such information is required of the damaged structure as well. One exception to this is a case where each sensor measures (or differences between adjacent sensors measure) local damage, in which case existence of damage implies location of damage.

We assume that the undamaged and damaged structures excited with an input u may be modeled as follows:

$$\begin{aligned} \dot{x}_u &= f_u(x_u, u, t) \\ y_u &= Cx_u \\ \dot{x}_d &= f_d(x_d, u, t) \\ y_d &= Cx_d \end{aligned}$$

or

$$\begin{aligned} \dot{x} &= f(x, u, t) \\ y &= [C \quad -C]^T x \end{aligned} \tag{1}$$

where x_u and x_d are the state vector of the undamaged and damaged systems, respectively, $x = [x_u \;\; x_d]^T$ and $f(x, u, t) = [f_u(x_u, u, t) \;\; f_d(x_d, u, t)]^T$. C is simply a matrix such that the measurable output y_u and y_d are some linear combination of the states. The goal of the excitation design is to maximize the difference between y_u and y_d, or, equivalently, to maximize y as defined in Eq. (1). Thus, the excitation design may then be stated as the following optimization problem:

$$\begin{aligned} &\max J = \frac{1}{2}\int_0^T y^T y \, dt \\ &\text{subject to } (1), \quad \|u\|_\infty < \gamma \end{aligned}$$

This may be rewritten as

$$\begin{aligned} &\max J = \frac{1}{2}\int_0^T x^T Q x \, dt \\ &\text{subject to } (1), \quad \|u\|_\infty < \gamma \end{aligned} \tag{2}$$

where $Q = [C - C]^T[C - C]$. If the input u is perturbed by u', the perturbed state trajectory is given by the tangent linear equation

$$\dot{x}' = A(t)x' + B(t)u' \text{ or } \mathscr{L}x' = Bu' \tag{3}$$

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where $\mathscr{L} = (d/dt) - A(t)$ and $A(t)$ and $B(t)$ are obtained by linearizing Eq. (1) about x and u. The resulting perturbation to J is then given by

$$J' = \int_0^T x^T Q x' \, dt \tag{4}$$

The goal of what follows is simply to re-express J' as a functional linear in u'. To that end, consider the tangent linear equation (3) integrated against a test function, r

$$\int_0^T r^T \mathscr{L} x' \, dt = \int_0^T r^T (\dot{x}' - Ax') \, dt$$

Using integration by parts, we can rewrite the above equation as

$$\int_0^T r^T \mathscr{L} x' \, dt = r^T x' |_{t=0}^{t=T} - \int_0^T (\dot{r} + A(t)^T r)^T x' \, dt = \int_0^T (\mathscr{L}^* r)^T x' \, dt + r^T x' |_{t=0}^{t=T}$$

where $\mathscr{L}^* = -(d/dt) - A(t)^T$. This is true for any test function, r. If we select r such that

$$\begin{aligned} \mathscr{L}^* r &= Qx \\ r(T) &= 0 \end{aligned} \tag{5}$$

then Eq. (4) can be rewritten as

$$J' = \int_0^T (\mathscr{L}^* r)^T x' \, dt = \int_0^T r^T \mathscr{L} x' \, dt = \int_0^T r^T B u' \, dt = \int_0^T \frac{DJ}{Du} u' \, dt$$

Thus,

$$\frac{DJ}{Du} = r^T B \tag{6}$$

Eq. (5) is referred to the as the adjoint equation. Therefore, given some initial guess at an excitation, u_n, Eq. (1) is solved for x. This x is then used to solve Eq. (5) in reverse time, since $r(T)$ is known. From r the gradient of the cost function with respect to the excitation, u, may then be calculated, and the next excitation is given by

$$u_{n+1} = u_n + \alpha \frac{DJ}{Du} \tag{7}$$

where α is found via a line search (e.g., Brent).

It is often the case that the difference in the output of the damaged and undamaged system can be increased simply by increasing the magnitude of the excitation. This is always true for linear systems, though not necessarily true for nonlinear systems. As a result, we force the magnitude of the excitation to be no greater than γ by setting

$$u_{n+1} = u_{n+1} \quad \text{if } \|u_{n+1}\|_\infty \leqslant \gamma$$

$$u_{n+1} = \gamma \frac{u_{n+1}}{\|u_{n+1}\|_\infty} \quad \text{if } \|u_{n+1}\|_\infty > \gamma$$

Thus, the iterative optimization of the excitation u is given by the following procedure:

(1) Select an initial excitation, u_n and solve Eq. (1).
(2) Solve Eqs. (5) and (6) to obtain DJ/Du.
(3) Obtain α from a line search.
(4) Calculate the next excitation u_{n+1} from $u_{n+1} = u_n + \alpha DJ/Du$.
(5) If necessary, normalize u_{n+1} such that $\|u_{n+1}\|_\infty \leqslant \gamma$.

This procedure is repeated until J converges (or some other stopping criteria is reached). It is worth noting that the conjugate gradient method was evaluated in addition to this standard gradient ascent. Because of the constraint on the excitation magnitude, the actual direction in an update step is not generally the same as the gradient. At least for the example below, this deviation in direction causes the standard gradient ascent to perform better than the conjugate gradient method.

3. Example

To demonstrate this technique, we consider the simple nonlinear system represented in Fig. 1. This system represents a 2 DOF system, with a nonlinear spring (which we take to be a cubic, stiffening spring). The excitation acts on the second mass, and we will take the only output to be the position of the second mass. The equations of motion for this system are

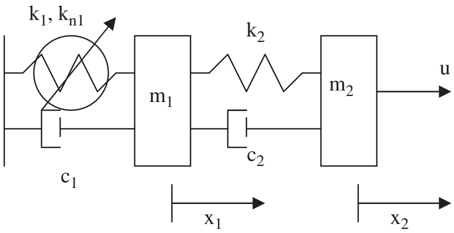

Fig. 1. Example 2 DOF nonlinear system.

then given by

$$\begin{aligned} m\ddot{x}_1 &= -k_1 x_1 - k_{nl}x_1^3 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) \\ m\ddot{x}_2 &= -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) = u(t) \\ y &= x_2 \end{aligned} \tag{8}$$

Damage is assumed to correspond to a decrease in k_{nl}. Such a scenario is representative of many systems that employ polymers, which are by nature nonlinear and subject to aging induced changes in their load-defection characteristics. The goal of the excitation design is to find a $u(t)$ that maximizes $y_u(t) - y_d(t)$, where y_u and y_d represent the outputs of the undamaged and damaged structures, respectively. Rewriting Eq. (8) as a set of first-order ODEs similar to Eq. (1) yields

$$\begin{aligned} \dot{x}_{1u} &= v_{1u} \\ \dot{v}_{1u} &= \frac{1}{m_1}[-k_1 x_{1u} - k_{nlu}x_{1u}^3 - c_1\dot{x}_{1u} + k_2(x_{2u} - x_{1u}) + c_2(\dot{x}_{2u} - \dot{x}_{1u})] \\ \dot{x}_{2u} &= v_{2u} \\ \dot{v}_{2u} &= \frac{1}{m_2}[-k_2(x_{2u} - x_{1u}) - c_2(\dot{x}_{2u} - \dot{x}_{1u})] \\ \dot{x}_{1d} &= v_{1d} \\ \dot{v}_{1d} &= \frac{1}{m_1}[-k_1 x_{1d} - k_{nld}x_{1d}^3 - c_1\dot{x}_{1d} + k_2(x_{2d} - x_{1d}) + c_2(\dot{x}_{2d} - \dot{x}_{1d})] \\ \dot{x}_{2d} &= v_{2d} \\ \dot{v}_{2d} &= \frac{1}{m_2}[-k_2(x_{2d} - x_{1d}) - c_2(\dot{x}_{2d} - \dot{x}_{1d})] \\ y &= x_{2u} - x_{2d} \end{aligned} \tag{9}$$

where k_{nlu} is the undamaged cubic spring coefficient and k_{nld} is the damaged cubic spring coefficient. Additionally, x_{1u} is the displacement of the first mass for the undamaged system, x_{1d} is the displacement of the first mass in the damaged system, and so on. The tangent linear equation is then given by

$$\begin{bmatrix} \dot{x}'_{1u} \\ \dot{v}'_{1u} \\ \dot{x}'_{2u} \\ \dot{v}'_{2u} \\ \dot{x}'_{1d} \\ \dot{v}'_{1d} \\ \dot{x}'_{2d} \\ \dot{v}'_{2d} \end{bmatrix} = A \begin{bmatrix} x'_{1u} \\ v'_{1u} \\ x'_{2u} \\ v'_{2u} \\ x'_{1d} \\ v'_{1d} \\ x'_{2d} \\ v'_{2d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \\ 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix} u$$

where A is given by

$$A = \begin{bmatrix} A_u & 0 \\ 0 & A_d \end{bmatrix}$$

and

$$A_u = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1 - k_2}{m_1} - 3k_{nlu}x_{1u}^2 & \frac{-c_1 - c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{c_2}{m_1} & -\frac{k_2}{m_1} & -\frac{c_2}{m_1} \end{bmatrix}$$

and

$$A_d = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} - 3k_{nld}x_{1d}^2 & \frac{-c_1-c_2}{m_1} & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{c_2}{m_1} & -\frac{k_2}{m_1} & -\frac{c_2}{m_1} \end{bmatrix}$$

Thus, using $Q = [C - C]^T[C - C] = [0\,0\,1\,0\,0\,0\,-1\,0]^T[0\,0\,1\,0\,0\,0\,-1\,0]$ and taking $m_1 = m_2 = 1$, $k_1 = k_2 = 50$, $c_1 = c_2 = 1$, $k_{nlu} = 10$, and $k_{nld} = 9$, we may follow the procedure specified in Section 2. It is worth noting that the damage is relatively minor (only a 10% change in a nonlinear term).

We start by picking an initial $u(t)$ equal to Gaussian noise with a peak amplitude of 10. Eq. (9) is then solved with the typical fourth-order Runge–Kutta method using a step size of 0.002 s. Note that over 30 s, $u(t)$ then has 15 000 points and thus 15 000 free parameters over which to optimize. Fig. 2 shows this initial random input and associated response. Clearly, if even a small amount of measurement noise was present, the damaged and undamaged responses would be indistinguishable with this initial random input.

The adjoint equation (5) is then solved to calculate DJ/Du from Eq. (6). The step size α is then obtained from a line search. The excitation signal is then updated according to Eq. (7). This process is repeated until convergence or some other stopping criteria is achieved.

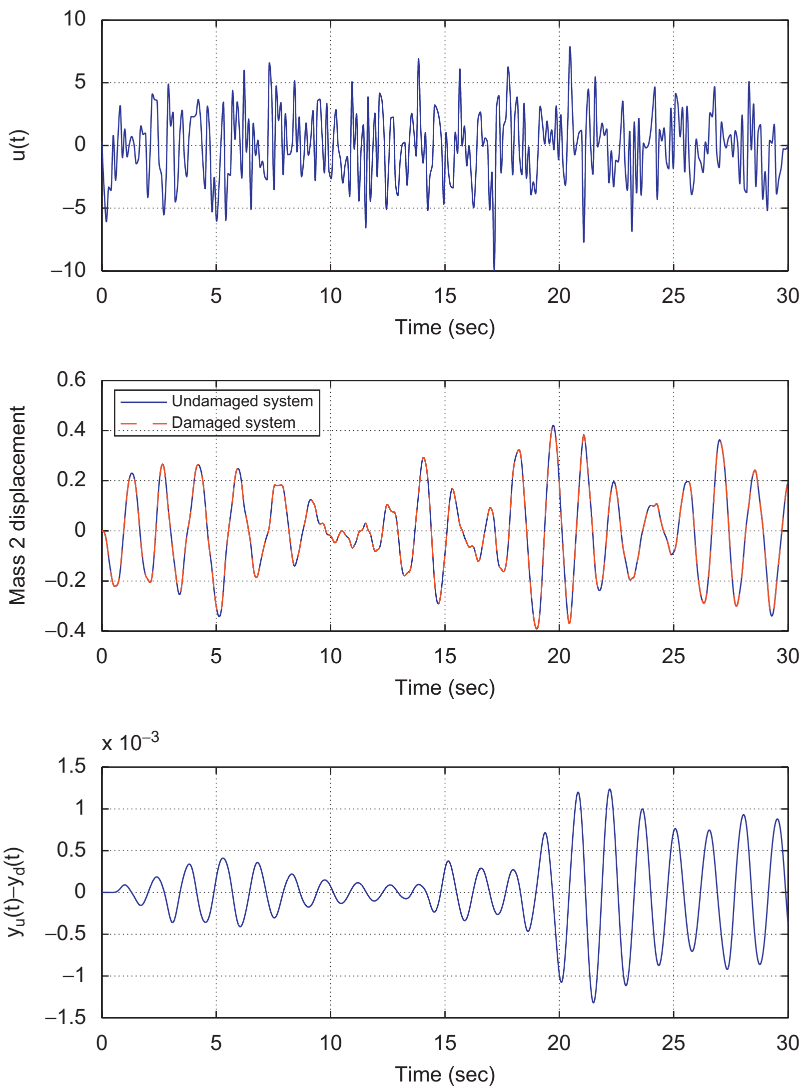

Fig. 2. Initial input and response.

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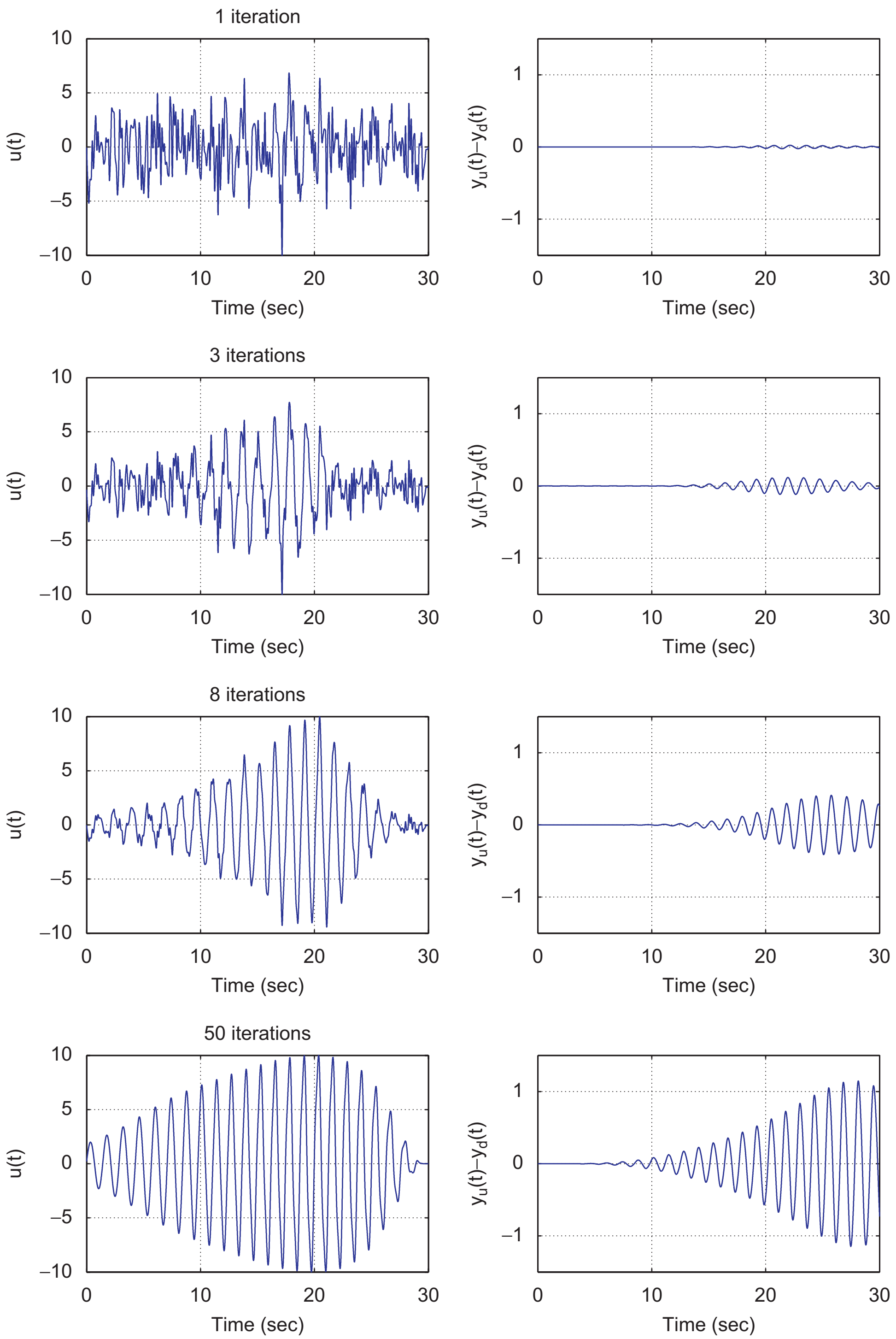

Fig. 3. Input and response at 1, 3, 8, and 50 iterations.

Fig. 3 shows the results of this process after 1, 3, 8, and 50 iterations, and Fig. 4 shows the cost function J as a function of iteration number. After 50 iterations, the input, constrained not to exceed a magnitude of 10, has increased the measurable difference between the undamaged and damaged systems by almost three orders of magnitude, even though the magnitudes of the responses of the undamaged and damaged system have only increased by approximately a factor of 8. There are now clearly identifiable differences in the actual system responses that would be visible even in the presence of significant noise, as evidenced in Fig. 5.

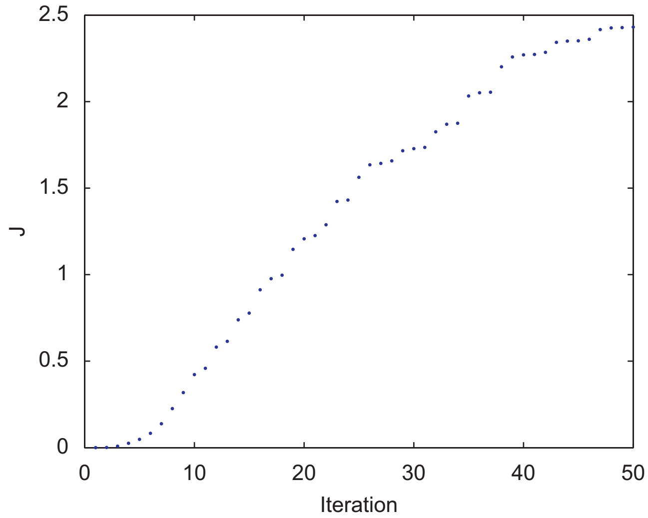

Fig. 4. Cost function evolution.

One might reasonably suggest that some other input, say a sinusoid or an impulse would have been a better input than random noise. Figs. 6 and 7 show the evolution of the excitation when a 0.6955 Hz sinusoid (the first natural frequency of the linearized system) and an impulse are used as initial excitations, respectively. Fig. 8 shows an approximately 3 s portion of the excitation produced from the three different initial excitations after 50 iterations. Clearly, all three different initial excitations are converging on what appears to be the same waveform, though, because this is a gradient ascent, no claim can be made that this waveform represents a global optimum.

4. Discussion

Not surprisingly, in the admittedly biased view of the authors this technique has many advantages over existing techniques. However, it is not without its disadvantages. Thus, this section attempts to highlight key advantages and disadvantages as well as to propose avenues of research to address the disadvantages.

One of the major advantages of this technique is its computational efficiency, relative to finite difference gradient-based approaches. Only two simulations are necessary to calculate a gradient, which is far better than a finite difference approach to gradient calculation, where the number of simulations necessary to estimate the gradient increases as the number of points in the excitation time history increases. For the example shown in this paper, the gradient calculation is approximately 7500 times faster using the developed technique than with a finite difference approach (two simulations compared to 15 001 simulations). To avoid this difficulty, other excitation design methods parameterize the excitation in an effort to reduce the number free parameters. This parameterization places artificial constraints in the form of the excitation. Consequently, the lack of constraints artificially placed in the form of the excitation represents another advantage of the developed technique.

An additional major advantage is the direct treatment of the ever-present limits on excitation magnitude. Because actuators always have a limited range, it is important for any excitation design technique to ensure that the actuator be capable of producing the designed signal. The technique developed here satisfies this important requirement. It is also worth emphasizing that constraints on actuator bandwidth are easily handled by incorporating actuator dynamics in the system model. In this case, the designed excitation is the signal sent to the actuator. It is also worth noting that there may be cases where it is desirable to place a constraint on the magnitude of the output of the system. This represents an area for future research.

Of the potential disadvantages, the first one likely to come to the mind of someone familiar with SHM is the necessity of a model of the damage, as this implies that the nature, extent, and location of the damage is known *a priori*. There are certainly applications where this information is truly unknown. However, for most applications, design engineers are able to predict likely failure modes, so the type(s) and location(s) of damage can be reduced to a manageable set. This information would allow excitations to be designed that are tailored to highlight the existence of damage at specific locations. In regard to the extent of damage, it should be noted that *all* structures are damaged. However, damage is a cause for concern only when it exceeds a certain level. Thus, the proposed technique is actually quite valuable in that it allows an excitation to be developed that will make some predetermined level of damage as "visible" as possible.

Another potential disadvantage, related to the first, is the extent to which modeling errors affect the excitation design. However, it is important to keep in mind that the excitation is designed to maximize the *difference* in the outputs of the

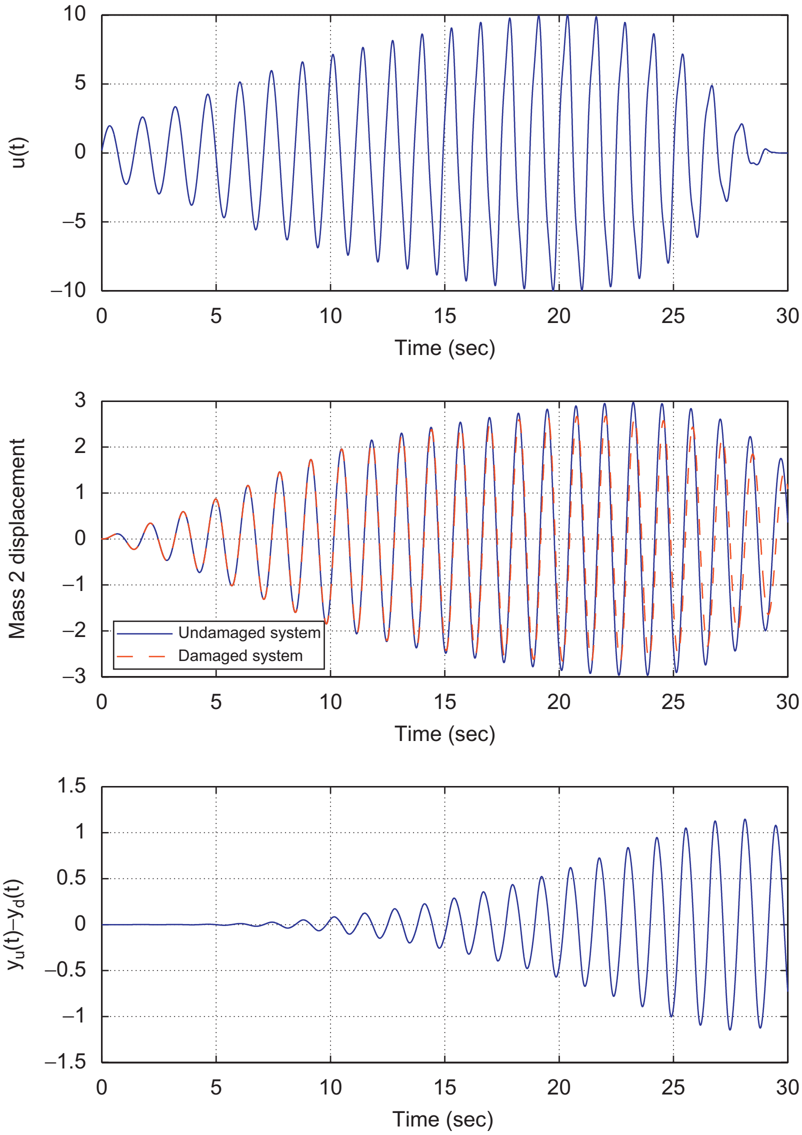

Fig. 5. Input and response at 50 iterations.

undamaged and damaged systems. Thus, modeling errors that are not specific to the damage itself appear in both undamaged and damaged models. Thus, there is reason to expect that the modeling errors will cancel themselves out to some degree. A rigorous treatment of the sensitivity of this technique to modeling errors is an area we are actively pursuing.

A third potential disadvantage relates to the formulation of the cost function. Some might object to a direct comparison (i.e., difference) of the time histories of the outputs. However, it is important to keep in mind the very general nature of Eq. (1). If instead of, say, displacement, one wanted to maximize the difference in some other feature, the functional relationship between the feature and the states could be included in $f(x, u, t)$, and the feature then becomes one of the states. Thus, one could develop excitations that would maximize the difference in various features. The sensitivity of the developed technique to the output or output feature is another area we are actively pursuing.

A final and somewhat obvious disadvantage pertains to all gradient based optimization techniques: the possibility of local extrema. While three significantly different initial excitations in the example appeared to be converging to a common waveform, there is no guarantee that this will always be the case, or that this waveform represents a global optimum. Nonetheless, the developed technique is very adept at taking some initial excitation and improving upon it. Thus, while the final excitation may or may not correspond to a global maximum, it will certainly be better than the original naive guess. Viewed from that standpoint, the method operates under a "first, do no harm" perspective.

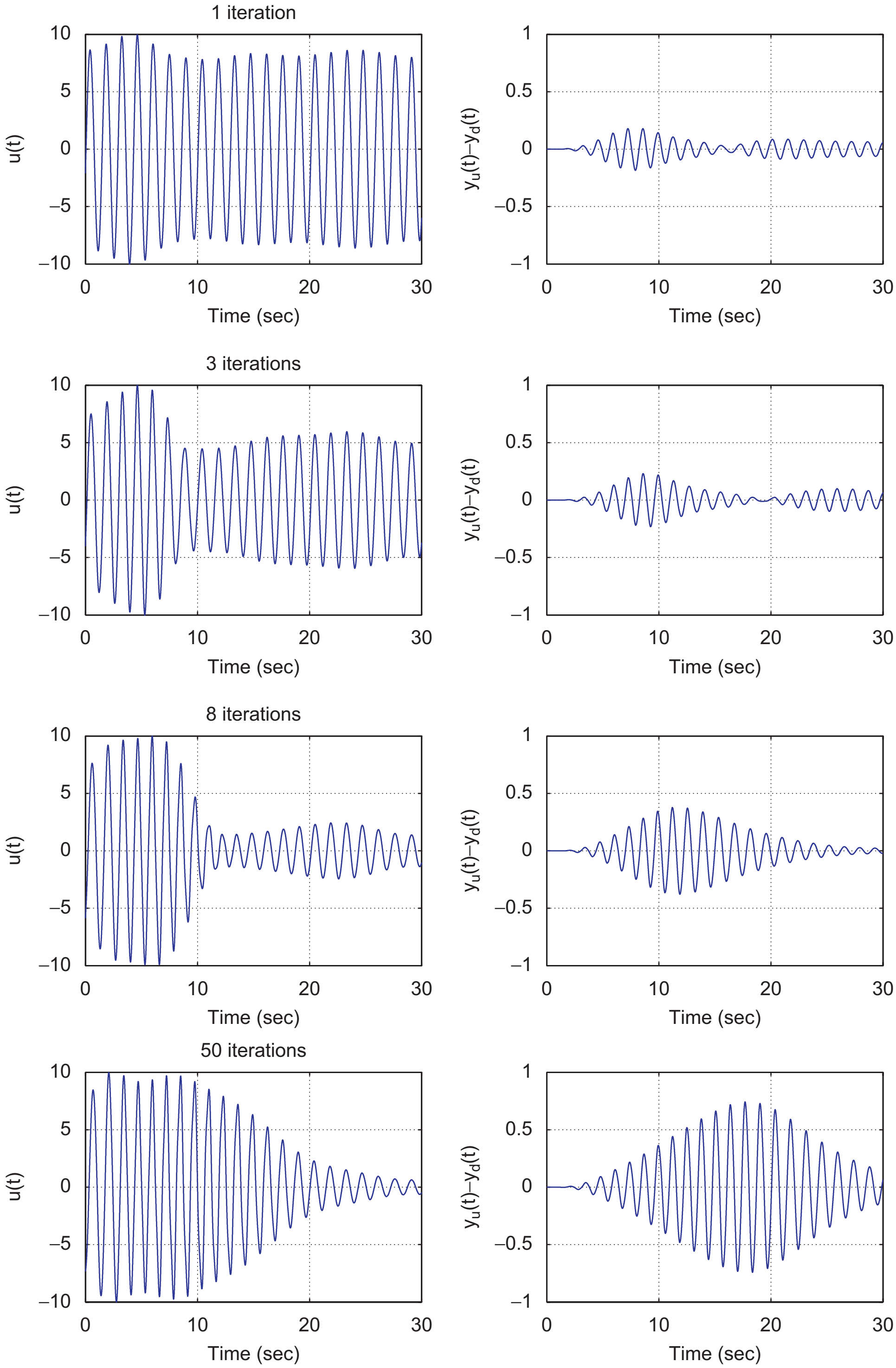

Fig. 6. Input and response at 1, 3, 8, and 50 iterations using an initial sinusoid.

5. Conclusions

A technique has been developed and demonstrated to optimize excitation for the purpose of enhancing the detectability of damage in a system. An augmented system is formed as the union of the undamaged and damaged systems. The difference between measurable outputs of the undamaged and damaged systems then simply becomes a state in this

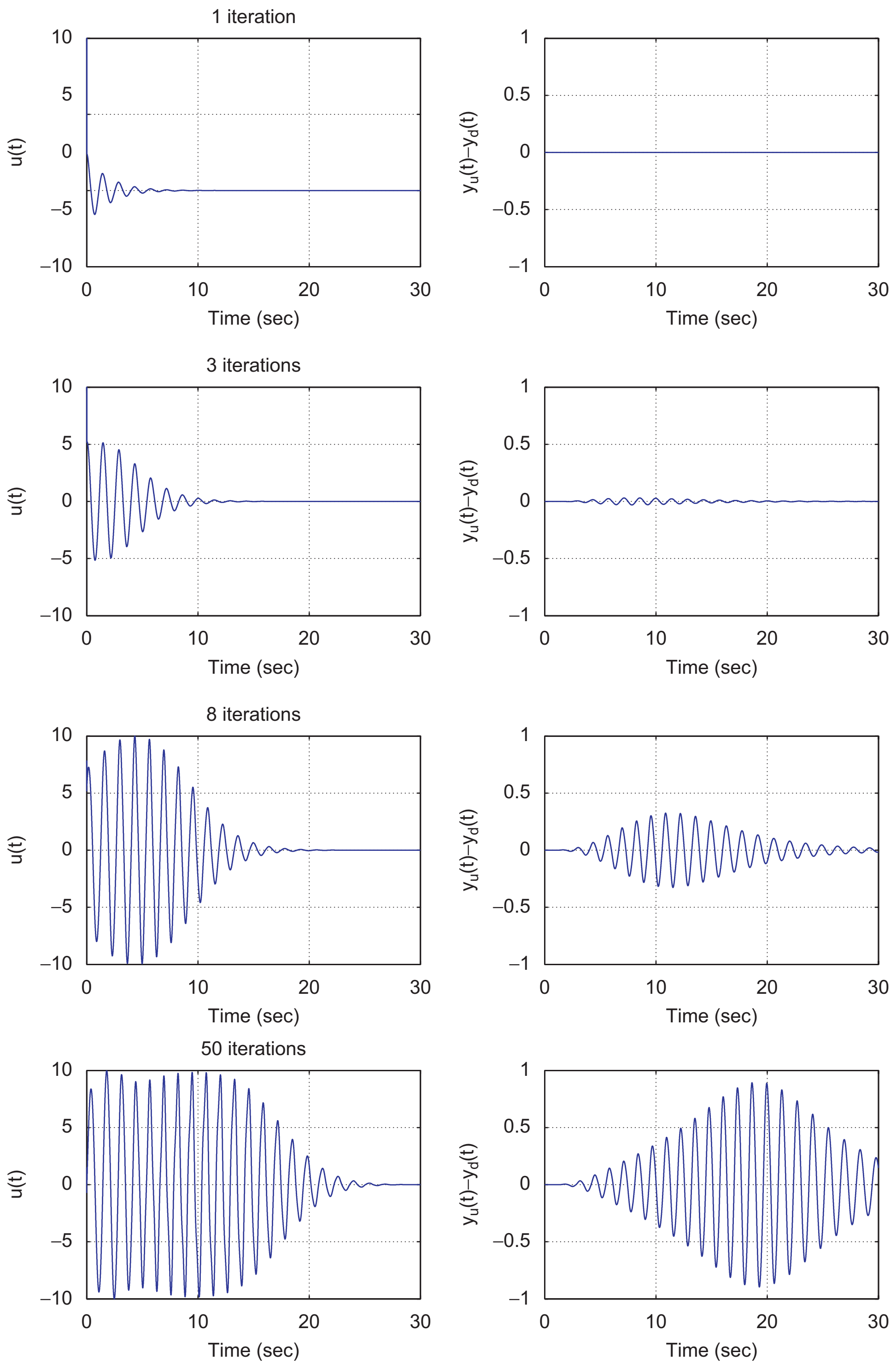

Fig. 7. Input and response at 1, 3, 8, and 50 iterations using an initial impulse.

augmented system which may be maximized by maximizing a related cost function. By formulating an adjoint version of the optimization problem, the gradient of this cost function with respect to the excitation may be calculated efficiently, and a normal gradient ascent procedure follows. The developed technique is quite general in that it makes very few assumptions on the nature of the system under consideration. The technique was demonstrated on a nonlinear 2 DOF system. Through the course of this demonstration, it was seen that the damage induced changes in the measurable

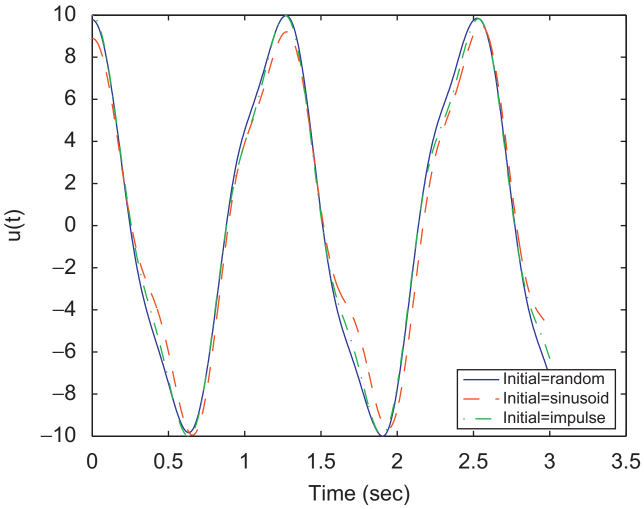

Fig. 8. A subset of $u(t)$ at 50 iterations.

outputs were increased by almost three orders of magnitude, even though the magnitude of the excitation remained unchanged.

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